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MOPP-I: AN OPTIMIZATION PACKAGE FOR MULTIPURPOSE BATCH OPERATIONS

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A number of algorithms have been developed -including enumeration of feasible production sequences, alternative task selection and the generation of alternative production lines- to determine the optimal sequence in which products and by-products should be produced and the times at which the various production operations for each product should be carried out to meet a given product demand pattern, taking into account the available equipment, storage costs, stopover penalties and other plant limitations.

Product interdependencies and utility requirements and constraints, which limit the number of operations that can be carried out in parallel, are also considered. These algorithms have been integrated into a single, efficient computer package that, integrated in presently available low-cost wokstations, is intended for routine use in industrial operational practice. A user-friendly menu-driven interface is also available.

The proposed methodology permits the solution to real, complex problems thanks to the use of a new, improved hybrid combination of heuristic and deterministic algorithms. Production planning in an already existing plant has also been succesfully achieved for short and medium-term production using problem decomposition techniques and integer-linear programming with restrictions. The algorithms have proved to be very fast when compared with existing deterministic methods. The industrial cases presented here demonstrate the practical use of the package under different plant production conditions.

Keywords: Batch processing, Multipurpose plants, Scheduling, Production Planning, Mixed-Integer-Non-Linear-Programming

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1. INTRODUCTION

Chemical plant production planning is traditionally perfomed according to prefixed nominal specifications such as production capacity, type and quality of raw materials and products, and demand pattern characteristics. The design of the plant itself often focuses as well on predetermined values of the parameters which specify the systems perfomance.

However, the real operating conditions of the plant are frequently quite different from the nominal specifcations. This problem is more pronounced in the case of batch processes, where there is a high degree of freedom available in assigning process tasks to various equipment and deciding when and how the products should be produced.

In addition, the economic and commercial situation in recent years has become quite unpredictable, thus contributing to increasingly irregular demand patterns for many chemicals. Sometimes demand forecasting has such a degree of uncertainty that profitability can be only achieved in very versatile plants, with enough flexibility to maintain certain production capacity levels within a wide range of products, and able to accept new products or changes in the demand patterns without significant further investment.

For these reasons, and the usually high value associated to the products obtained, batch and semicontinous processes with multiproduct and multipurpose capabilities will continue to be a prominent and permanent feature of a large sector of the processing industry that will give a suitable answer to the following market and production requirements:

- Variability in the supply of raw materials
- Manufacturing capability for products requiring the use of the same equipment
- Instability of products and intermediates
- Variability in product demand patterns

Although a number of computer-aided methods for the simulation, design and operation of continous plants have been developed in recent years, the planning and design of batch operations has not received proper attention, mainly due to the greater complexity of this problems, having many more degrees of fredom that of a continous operation, even for a single product. When several products are present, the potential variations in the operation of the batch plant increase considerabily depending on:

- Number of products and interrelationship
- Demand pattern for each product

- Available equipment
- Task assigned to each production unit
- Configuration of the production lines

Batch plants can be classified according to each of the above characteristics as follows:

- Manufacturing plants for a single product.
- Multiproduct plants with or without interrelationships between products: Production lines are always the same for the same product.
- Batch plants with deterministic demand pattern.
- Batch plants subject to demand uncertainty.
- Flexible batch plants:
  - . by the type and number of available process equipment (batch, semicontinous).
  - . by task assigned to equipment (fixed or flexible).
- Multipurpose plants: production lines may be diverse for the same product.
- Multiplant: multiproduct plant structures working in parallel.

The actual plant may be into one or several of the above categories, and the degree of complexity at the design stage will depend on the desired category; essentially, the complexity increases with higher flexibility requirements. Obviously, the degree of development of available design tools is directly related to the flexibility intended in the final design.

2. PREVIOUS WORK

Optimizing the design of batch plants is a subject of increasing interest in recent years after decades of careless oversizing. It was not until the sixties that the problem of minimizing investment costs in multiproduct plants began to attract some atention. Initial work considered equipment sizing and processing times as variables to be included in the optimization function (1, 2). An algorithm was proposed to minimize equipment investment under specific restrictions, which was later extended to accept the multiproduct case (3, 4). However the simplicity of this approach was not suitable to real plant situations. Later, Sparrow et al. (5) developed the first systematic tool for multiproduct plant design (MULTIBATCH package).

More recently, non-linear programming methods were used to optimize the integer variables involved (6), which were extended later to include the multipurpose case (7). Production planning for selected dominant campaigns was studied by Mauderli and Rippin (8, 9). Variable demand patterns were included and cycle time restrictions were also introduced to obtain the production planning. Additional restrictions due to limitations in utilities and product instability were also considered in later work (10).

The problem of storing intermediates is becoming a subject of increasing interest as a way to decouple processing units and thus optimize production times and capacities, leading to minimum investment costs (11, 12, 13). Dynamic programming is used to solve the problem when integer values are found in the calculation of cycle times or production capacity.

The interaction between continous and discontinous sectors of the same plant has also been studied. The solution to this problem is attempted by introducing intermediate storage subjected to variable flow of material (14, 15).

Linear programming techniques have been widely used to solve the problem of intermediate storage, with specific applications to the petrochemistry industry (16). Linear programming subjected to restrictions, combined with dynamic programming, have been used to find the optimum storage for the multiproduct case in the presence of uncertainty in the demand and risk of equipment failure (17, 18).

Karimi and Reklaitis (19, 20) have considered intermediate storage as a decoupling device between serial systems. Stochastic variations are introduced in all processing parameters: initial processing times, cycle times, batch size and flow of material. Knopf (21) considers branch and bound techniques with process simulation to determine the optimum sequence for the problem consisting of a single processor and its input queue followed by N units in parallel of intermediate storage and then P units of process equipment. The Johnson algorithm (22) is appropriately modified to determine the upper limit with excellent results.

Although specific software is emerging aimed for use in practical cases (23), most of the work done has been restricted to the academic environment with little impact -if any- in the industrial world. Present trends indicate that significant effort is needed to develop the kind of practical tools required for immediate use in plant design practice and operation of batch processes. In this sense the software package presented here has been conceived as a practical tool intended for routine use in industrial practice.

3. PRODUCTION PLANNING IN MULTIPURPOSE PLANTS

Production planning is a subject of major concern in the batch processing industries. Product 'recipes' are subjected to frequent variations due to increasingly irregular seasonal fluctuations in the demand patterns. Although systematic methods for production planning have long been the subject of extensive research work (19, 24, 25, 26) the results obtained have been of limited use in industrial practice. In this work a practical software system has been developed to obtain the short and medium-term production planning for the most complex case of a multipurpose facility subjected to real operating constraints.

Production planning in batch processing depends essentially on the number and type of products to be produced, subjected to a demand which varies with time. Some products will have characteristics that require them to be made separately in the production line, but some others will be interrelated as intermediates for final products and subproducts. The demand distribution over medium and long time periods will determine the amount of final (and eventually intermediate) products and raw materials required.

The following premises are implied:

- The number, type and quantity of products to be manufactured are determined according to market considerations and following the company's policies.
- Adequate operating conditions and the tasks required to manufacture the products are defined. Equipment production capacity requirements for each product are also determined.
- The demand pattern is known.

4. SOLUTION METHODOLOGY

The system to be considered for short and medium term production planning takes into account the production and storage facilities from the raw material supply through the delivery of final products to the customer. The solution methodology consists of:

- Study and characterization of batch plants and processes
- Enumeration of feasible production sequences
- Selection of dominant production lines
- Task sequencing
- Optimum production planning with restrictions

CHARACTERIZATION OF BATCH PLANTS AND PROCESSES

The available equipment in multipurpose plants ca be used for several tasks and some tasks may require a wide range of equipment modules, thus allowing for a high degree of flexibility. The equipment may be grouped according to the standard unit operation perfomed. Within each group, the specific characteristics of each type of equipment may be considered. Table 1 lists the equipment commonly available in batch plants indicating the variable that best represents its production capacity.

The maximum production capacity R_{ijn} of product i for task j in the equipment n is given in terms of the capacity factor S_{ijn} or else the equipment size needed at that stage per unit amount of final product.Therefore

$$R_{ijn} = \frac{\text{Capacity of Equipment } n}{S_{ijn}} \tag{1}$$

When semicontinous equipment is considered the capacity factor depends on the equipment, task and product (S_{ijno}) and the processing time T_p

$$S_{ijn} = \frac{S_{ijno}}{T_p} \tag{2}$$

Residence times for batch and semicontinous equipment are given by the expressions

$$T_r = T_0 + T_1.C_p^a \qquad \text{for batch} \tag{3}$$

$$T_r = T_0 + T_1.F_p^a \qquad \text{for semicontinous} \tag{4}$$

where a, T_0 and T_1 are constants for each equipment type l product processed i and task perfomed j, respectively.

Let B_{ijn} be the amount of product i manufactured in the equipment $n/n \in K_j$ = equipment set used for task j. Then, the residence time for each task j and type of equipment n may be calculated by the expression

$$T_{ijn} = T_{oijl} + T_{1ijl} \cdot B_{ijn}^{a_{ijl}} \quad \forall n \in \text{ class of equipment } l \tag{5}$$

with the constraint

$$T_{ij} = T_{ijn} \quad \forall n \in K_j \tag{6}$$

For batch equipment production times and residence times are the same $T_p = T_r = T_{ijn}$.

For semicontinous equipment the following expression applies:

$$T_{ijn} = T_{oijl} + T_{1ijl} \quad (B_{ijn}/T_p)^{a_{ijl}} \tag{7}$$

where T_{oijl}, T_{1ijl} and a_{ijl} are specified constants for each product, task and type of equipment. Values of a_{ijl} for standard equipment are given in Table 2.

To determine the maximum production capacity and processing times for each task, the algorithm shown in Figure 1 has been developed.

Once production capacity and operating times for each task have been obtained, the utilities demand with time is also calculated using the following expression:

$$G_{kijn} = G_{okijl} + B_{ijn} \cdot G_{1kijl} \quad \forall\, n \in \text{ class of equipment } l \tag{8}$$

where G_{kijn} is the demand of utility k at stage j for the equipment type n and G_{okijl}, G_{1kijl} are constants associated with product i.

The utilities demand, when processing with semicontinous equipment, may be considered as uniform, as in continous operation. When batch units are involved, the utilities demand should be specified piecemeal during batch operations.

ENUMERATION PROCEDURE AND SELECTION OF 'DOMINANT' PRODUCTION LINES

Once the time-capacity requirements in each equipment module are known for every product, all possible production variants are generated by an enumeration procedure that takes into account the possibility of available equipment working in parallel, initial and final task overlapping and instability of intermediate products. Non-feasible sequences are progressively eliminated so that only favourable candidates are subjected to full evaluation.

Dominant production lines are selected by heuristic rules based in the following parameters

$$P_1 = \frac{\text{Capacity}}{\text{Total Processing Time}} \tag{9}$$

A second parameter P_2 is calculated as follows:

$$P_2 = \frac{1}{n_t} \sum_{j=1}^{n_t} \sum_{n \in K_j} (R_{ijn} - B_{ijn})^2 \tag{10}$$

Thus P_2 indicates the idleness of the equipment used in the production line to elaborate product i. For semicontinous equipments $R_{ijn} = B_{ijn}$ as it should be obvious.

A third parameter P_3 is defined as

$$P_3 = \frac{\text{Overall Production Cost}}{\text{Processing Capacity}} \tag{11}$$

The best candidates are selected for maximum values of P_1 and P_3 and minimum P_2, or else to maximize

$$\log P_4 = a_1 \log P_1 + a_2 \log P_2 + a_3 \log P_3 \tag{12}$$

The values of a_1, a_2 and a_3 may be specified by the user according to specific problem characteristics and the desired objective function, and default values are provided by the system ($a_1 = 1$, $a_2 = a_3 = -1$). The flowchart of the enumeration procedure and the selection of dominant production lines, after calculation of the previously defined heuristic parameters, is shown in Figure 2. The detailed step by step procedure is given in reference (27). Typical computing times used by this algorithm in a test case are shown in Table 3.

SCHEDULING

The problem of scheduling several independent products on different processing equipment of multipurpose characteristics within a prefixed time interval is a complex optimization problem that has received considerable attention in the literature. This is because some of the very basic problems in scheduling have never been solved in an acceptable fashion when large size and conflicting problems, as those encountered in the chemical industry, are present, yet the literature is full of solution attempts. There is also the fact that since no generally applicable analytical solution exists, many potentially valid approaches can be taken to the problem.

The solution methods range from analytical to heuristic algorithms and simulation. The integer programming technique (28) restricts the values of the variables to 0 or 1. If we want to consider a great number of technologically possible sequences, the number of variables increases considerably, thus limiting the applicability of this technique. The "branch and bound" approach (29) minimizes the analysis by cutting off as many branches of the search tree as possible. Still the effectiveness of this method is still limited to small-scale problems (31, 32, 33).

In this work an heuristic strategy is used to approximate the optimal solutions using a realistic approach without excessive loss of accuracy. The objective of this strategy consists in minimizing the makespan problem (29). The sequencing algorithm considers the posibility of several equipment modules running in parallel (in phase), the existence of unstable conditions and task overlapping. In addition, manpower and utility limitations are also taken into account.

Priority rules MWKR (Most Work remaining) and SPT (shortest processing time) are used to deal with equipment in parallel taken as a single process unit and then calculate the processing times for all modules involved. The same strategy is used for those tasks essentially interrelated due to the presence of unstable products. Priority rules MWKR, SPT and LWKR (Least Work remaining) are used to evalue a "theoretical" task requiring the processing time equivalent to that needed by the sum of the individual tasks associated to unstable products. Utilities manpower and stopover limitations are the final criteria for selecting the adequate task each time interval between those showing similar values of priority rules. A step by step description of the algorithm is given in reference (27). The simplified scheduling algorithm flowchart is given in Figure 3. CPU time required by this algorithm in the test problems is shown in Table 3.

PRODUCTION PLANNING WITH RESTRICTIONS

Production planning addresses the problem of finding the best processing strategy to obtain the desired products from a variety of raw materials, under internal (utilities, manpower, intermediate products) and external (market demand and forecasting) restrictions.

The multipurpose batch plant represents the most complex processing facility, in which there is no common pattern of movement for the products and successive batches of the same product may take different routes through the process. In this work, the most general case is considered: when two or more products can be produced simultaneously; new products are added throughout the process and other intermediate products may pass onto the other sections of the plant (Figure 4).

It will be assumed that the selected processing lines ($m = 1, .., n_b$) operate at their nominal capacity (X_{im}). This assumption is even more realistic when the number of batches (N_{imt}) involved increases, since the penalty cost incurred by overproduction will be irrelevant in the long term with respect the total production cost. Additional assumptions are that the inventory (h_i) and production (C_{im}) costs are practically time independent. Then the function to be minimized will be

$$\begin{aligned} z = &\sum_{i=1}^{n_p} h_i \sum_{t=1}^{n_t} (n_t - t)\ D_{it} + \\ &+ \text{Min}\left\{\sum_{i=1}^{n_p} \sum_{t=1}^{n_t} \sum_{m=1}^{n_b} [h_i\ (n_t - t) + C_{im}]\, X_{im} N_{imt}\right\} \end{aligned} \tag{13}$$

subjected to the condition that the production up to certain period must be greater than or equal to the demand (D_{it}) in this period and its precedings.

$$\sum_{t=1}^{t_1} \sum_{m=1}^{n_b} X_{im} \; N_{imt} \geq \sum_{t=1}^{t_1} D_{im} \qquad t_1 = 1, \ldots, n_t \qquad i = 1, \ldots, n_p \tag{14}$$

where $N_{imt} > 0$ and always integer.

Product interdependence is taken into account by introducing the variable g_{ii_1} or the required amount of product i to obtain i_1, thus leading to the following expression

$$\begin{aligned} z = {} & \sum_{i=1}^{n_p} h_i \sum_{t=1}^{n_t} (n_t - t) D^*_{it} + \\ & + \text{ Min } \left\{ \sum_{i=1}^{n_p} \sum_{t=1}^{n_t} \sum_{m=1}^{n_b} \left[C_{im} + \sum_{i_1=1}^{n_p} h_{i_1} (n_t - t) f_{ii_1} \right] X_{imt} N_{imt} \right\} \end{aligned} \tag{15}$$

with the constraint

$$\sum_{i_1=1}^{n_p} \sum_{t=1}^{t_1} \sum_{m=1}^{n_b} X_{i_1 m} N_{i_1 mt} f_{ii_1} \geq \sum_{t=1}^{t_1} D^*_{it} \qquad i = 1, \ldots, np \qquad t_1 = 1, \ldots, n_t \tag{16}$$

where

$$f_{ii_1} = 1 \qquad \forall\, i = i_1$$

$$f_{ii_1} = -g_{ii_1} \qquad \forall\, i \neq i_1$$

and D^*_{it} is the market demand of product i for the period t, which is related to D_{it} as follows

$$D_{it} = D^*_{it} + \sum_{\substack{i_1=1 \\ i_1 \neq i}}^{n_p} \sum_{m=1}^{n_b} X_{i_1 m} \; N_{i_1 mt} g_{ii_1} \tag{17}$$

The problem as formulated above will have $n_p \times n_b \times n_t$ integer variables with $n_p \times n_t$ restrictions. It should be obvious that the high dimensionality implied even for a small number of products compels one to find efficient solutions to solve practical cases. In this work the solution strategy includes the decomposition of the original problem into n_p subproblems of the form

$$z_i = \text{Min} \sum_{t=1}^{n_t} \sum_{m=1}^{n_b} [C_{im} + h_i(n_t - t)] \; X_{im} \; N_{imt} \tag{18}$$

with the constraint

$$\sum_{t=1}^{n_t}\sum_{m=1}^{n_b} X_{im}\, N_{imt} \geq \sum_{t=1}^{n_t} D_{it} \tag{19}$$

Then n_p subproblems are solved in such a way that those involving products i not required by i_1 are solved first. This strategy dramatically reduces the dimensionality of the problem and hence simplifies the calculation procedure.

Several integer programming techniques have been considered to solve the production planning problem in the form

$$\text{Min } \underline{c}\,\underline{n} \tag{20}$$

with the constraints

$$\begin{array}{ll} \underline{B}\,\underline{n} \geq \underline{d},\ \underline{b}_t\underline{n} \geq d_t & t = 1,\dots,n_t \\ n_{mt} \geq 0,\ \text{integer} & mt = 1,\dots,n_t \times n_b \end{array}$$

where the non-zero elements of the restriction matrix $\underline{B}(n_t, n_t \times n_b)$ are $n_b n_t(n_t + 1)/2$, over a total of $n_t\, n_b$ elements.Therefore the occupation percentage level will be

$$\frac{n_b n_t(n_t + 1)/2}{n_t(n_b n_t)} = \frac{n_t + 1}{2n_t} \tag{21}$$

which tends toward 50% when n_t is large enough

A modified Branch and Bound technique with integer variables has been used satisfactorily, leading to close to the optimal solutions -within 2% accuracy with respect to the continous solution- in very short computing times (see Table 3), thus comparing favourably with alternate methodologies (30).

COMPUTATION OF AN OPTIMAL PRODUCTION PLAN

The setting up of an optimal production plan requires an integrated approach in which task scheduling and inventory strategies must be optimized together, taking into account actual plant limitations, to obtain realistic solutions in industrial practice. Therefore, the following batch processing restrictions must be also considered:

- Processing time: tasks assigned to each period must be accomplished within preset time intervals.
- Utilities: nominal plant specifications must be met (steam, electrical power, manpower).

. Storage facilities: for final and intermediate products, raw materials and by-products.

The solution to this problem has been attempted introducing proven heuristic rules that simplify the calculation procedure by restricting the values of the integer variables. The first limitation comes from the production capacity of each production sequence, obtaining a single product within the available processing time in each period (production line capacity factor). When several products are considered a preferential order must also be established wich depends essentially on the storage and production costs. Since production in each period is determined basically by product demand within the same period, limiting conditions will influence previous production time. Thus it may be necessary tho shift backward certainn operations to meet the production requirements. This means that additional storage charges will be incurred for earlier production. This additional cost will depend on:

. the product itself: each product will have different inventory costs.

. the production line capacity factor.

. the number of operations to be shifted and times required.

To determine the limiting variable, the parameter P_5 is evaluated for each product

$P_5 =$ (storage unit cost) × (Max. number of tasks per period and production line) × (production capacity)

The limiting variable within each period will have the minimum value of P_5. In the determination of the limit itself that will insure the optimum production plan, enumeration procedures have been avoided; instead, the following algorithm is proposed (LIM):

Step 1 Given the production sequence time T_s and the available processing time in each period T_u , calculate the operation time to be shifted backwards.

Step 2 Determine P_5 for each variable and select the limiting variable.

Step 3 Evaluate the number of production sequences (N) to be shifted.

Step 4 Solve the integer problem with the new restriction added.

Step 5 Task scheduling and calculation of $T_{s_{new}}$. Check if $T_{s_{new}} < T_s$ and maintain this restriction.

Step 6 Check if $T_{s_{new}} < T_u$, otherwise go to step 2.

The global algorithm proceeds as follows:

Step 1 Input data and initialization: market demand pattern for each product, products recipes, available equipment and utilities restrictions. Input data is introduced via user-friendly menus.

Step 2 For each product and production line, calculate the maximum allowed number of tasks within each period.

Step 3 Production planning for the first product taking into account product preferential order and product interrelationship.

Step 4 Check if, for a given product, the maximum number of tasks incurred in the production planning can be perfomed; then go to Step 8.

Step 5 Otherwise, obtain parameter P_5 for this product and determine the limiting variable.

Step 6 Use the procedure described before (LIM) to evaluate the constraint to be used.

Step 7 Production planning for this product. Go to step 4.

Step 8 Go to step 4 and continue with next product in the preferencial order. After last product continue with step 9.

Step 9 Determine the global objective function for the first solution found.

Step 10 If no additional restrictions have been found for any product go to step 18.

Step 11 If for each type of production line, production can be achieved when assigning the maximum values to all variables implied in elaborating the same product, go to step 18. Otherwise, do steps 12 to 18 for all periods following a backwards procedure.

Step 12 If all tasks assigned to a given period can be perfomed, go to step 17.

Step 13 If the period analyzed is the first, and overall demand requirements cannot be met, the period is fathomed. Then go to step 21.

Step 14 Find P_5 for all products and production line types. Determine the limiting variable and product affected.

Step 15 Find the value of the constraint by using the LIM algorithm.

Step 16 Production planning -taking into account the additional constraint- for all products affected by this constraint following the preferential order established. Go to step 12.

Step 17 If steps 12 through 16 have been checked for all periods continue with step 18. Otherwise go to step 12 and continue with the next period in backwards order.

Step 18 Determine the global objective function.

Step 19 Examine the results obtained for the desired periods.

Step 20 Determine inventory and demand levels for each period.

Step 21 Output. Interactive graphic system is used.

The simplified global algorithm for the optimal production planning is given in Figure 5.

5. THE COMPUTER PROGRAM MOPP

The computer programm MOPP (Multipurpose plant Optimal Production Planning) has been developed to solve the problem of optimal production planning for batch multipurpose plants based on the calculation procedures described before.

The program was initially written in extended FORTRAN 4 and FORTRAN 77 for a Digital Vax 780 computer. A new version for the IBM computer is currently available, and more recently the program has been implemented in the SUN M/52 workstation provided with interactive facilities. A friendly menu is also available to ease the interface with the user, in this case it is the plant engineer or the production manager to whom this tool is particulary addressed. The main core of the program contains about 10.000 lines; another 10.000 lines will constitute the interface with the user. Actual dimensioning of the program variables is preset by the user.

The program structure appears in Figure 6. Indicative computing times are shown in Table 3 which demonstrates the significant perfomance of the different algorithms implied. A commercial version with User Manual guide is currently available and additional information is available from the authors.

6. INDUSTRIAL APPLICATIONS

The software package developed has been tested in several plant facilities. The following example will illustrate the use of this computer program in industrial practice.

The case study chosen belongs to a series of tests carried out in a Pharmaceutical Plant located near Barcelona, Spain. A section of the plant, producing intermediate materials to be further transformed into final products, has been selected to simplify the presentation of the calculation procedure. Three basic products are manufactured which are characterized by seasonal variations in their demand patterns. Because of the high added value of the materials to be produced it would be of primary importance to reduce the storage costs.

The plant consists of a typical multiproduct and multipurpose facility, where the same equipment can be used to obtain the different products, assuming the appropriate cleaning is perfomed. A total of 17 pieces of equipment are available for production, which may be grouped into 4 types: Reactors, Heat exchangers, filters and dryers. Table 4 indicates the characteristics of each piece of equipment. Although the values shown in the Table are consistent, they differ from the actual plant values to protect the confidentiality of the results obtained.

Three of the reactors (REAC RF) are equipped with reflux condensers and heating systems. There is a stirred tank (REAR PR) specially designed for pretreatment of raw materials, and two additional stirred tanks (REAC CH) that can be used as heat exchangers; the three units are equipped with heating systems. Three heated decanters (REAC DC) are also available to separate suspended solids. Two types of filters can be used; two units of the vertical type (FILT VX) with a slow filtration rate (0.02-0.05) $m^3/h\ m^2$) for low solid concentration suspensions (1-10%), and two units for the horizontal type (FILT HX) for medium solid concentrations (10-20%). All dryers are double-cone-rotary-type dryers (DRYE DC) and are vacuum operated and steam heated.

Interconnection between the different equipment is completely flexible.

The available utilities are:

S1: Steam with a limiting production power of 840 MJ/h (233 Kw)

S2: Cooling system with a limiting production power of 480 MJ/h (133 Kw.)

PRODUCT DESCRIPTION

The section of the plant under study produces three products which will be called A, B and C. The manufacturing process for each product is indicated in Figures 5, 6 and 7. None of the products are used as intermediates for the others.

To produce product A, raw materials are introduced into a stirred tank reactor where they react at 373 K in reflux conditions (Figure 5). The solution obtained is then cooled to 333 K and diluted with a new reactant up to 50% to yield a white precipitate. The supernatant is removed to be used as reactant in another section of the plant. The precipitate is then filtered, dryed and stored, while the water coming from the filter press is decanted, neutralized and disposed. The product 'recipe' is given in Table 5, which includes the actual values obtained from the factory and other information required in addition to the plant and process characteristics indicated before. Although presently a user friendly menu simplifies the handling of input data, for the purpose of this presentation the data collected in Table 5 uses specific codes to indicate task overlapping. Thus, the 'Type of Equipment' also indicates the type of product associated, coded as follows:

< 0 batch equipment
> 0 semicontinous equipment
$= 1$ unstable intermediate product
$\neq 1$ stable intermediate product

and the connection between tasks ('overlapping Type') is given in the form:

< 0 means task continuation
> 0 means task precedence
$= 10$ means overlapping time (time units)
$= 20$ means previous task overlapping time in %
$= 30$ means actual task overlapping time in %

The manufacture of product B requires the following processing steps (Figure 8). Some raw materials are introduced into a stirred tank at low temperature (273 K), while other reagents are diluted in another similar tank. The contents of the first reactor is added to the second and the mixture obtained is heated to 253 K. Several reactions take place at this temperature, then the mixture is cooled to ambient temperature (285 K) and neutralized. After concentration and drying under vacuum, the product B in solution is obtained via solvent extraction. This solution is decanted and the product precipitates by adding a new reagent to be finally dryed. The supernatant is also decanted and precipitation occurs again by adding another reagent while the remaining solution is recycled. The suspended solids are finally filtered to go on to later purification stages. Table 6 gives the details of the 'recipe' for product B, complemented with the additional data required to produce the production planning as before.

Figure 9 indicates the processing steps for product C. Here, some raw materials are pretreated in a reactor with reflux. Upon the addition of a new reagent a series of reactions occur.

The final mixture is cooled to ambient temperature thus obtaining an intermediate product in suspension which is separated by filtration. After drying, the intermediate product reacts with other raw materials in a stirred tank with reflux. Suspended solids are filtered and the clarified solution is dryed, dissolved and cooled to 272 K. This suspension is then filtered and the product, once dryed, is conveniently stored. Table 7 contains the 'recipe' for product C and additional data as before.

PLANT OPERATION

The plant described earlier operates 10 hours a day for 5 days per week. There is also an annual four-week maintenance period. The total annual production time of the plants is therefore 48 weeks of 5 days each week and 10 working hours per day.

The long-term production occupies the calendar year of 52 weeks and the medium-term production comprises a four-week period.

The demand pattern for each product and period throughout the calendar year is presented in Table 8, where the amounts shown are given in kilograms. Period 13 is stopover for maintenance.

ALTERNATIVE PRODUCTION LINES

The enumeration of alternative production lines and subsequent selection of the 'dominant' production lines (module BATCH) is summarized in Tables 9, 10 and11 for the three products. The selection creiterion has been

$$\text{Max } P_4 = P_1 \ / \ (P_2 \times P_3)$$

A total number of 229 alternatives have been analyzed for product A, only 18 for product B and 204 for product C. The small number of possible alternatives to produce B is mainly due to the fact that the same type of equipment (reactor-decanter) is required for two interrelated tasks (overlapped tasks) which reduces considerably the number of possible equipment combinations.

The Gantt chart, corresponding to production line 1.153 to produce product A, shows the operation times of the plant equipment involved and the utilities required (MJ/min) throughout the manufacturing process (Figure 10) The alternative 2.10 for product B is shown in Figure 11, and the Gantt chart representing the production line 2.15 of product C is given in Figure 12.

PRODUCTION PLANNING

The MOPP software package has been used to produce the best production planning for the plant that will meet the demand pattern required for each product (Table 8) at minimum cost (Equation 13). Market prices for each product are

1.500 ptas/kg of product A

2.300 ptas/kg of product B

2.000 ptas/kg of product C

Inventory costs are calculated to be 2% of the product selling cost for each medium-term period, or

30 ptas/kg per medium-term period in the case of product A

46 ptas/kg per medium-term period in the case of product B

40 ptas/kg per medium-term period in the case of product C

Initially, the optimum production planning was obtained for the case when there are no restrictions on the available utilities. The value attained by the optimization function was 13.607.700 ptas. and the computing time required was 4 minutes and 24 seconds (using a Digital Vax 780 Computer). It is interenting to observe the inventory distribution obtained (Table 12) when compared with the results obtained under utilities restricted within the available production limits (233 Kw of steam power and 133 Kw of cooling power) that are shown in Table 13. The value of the optimization function in the latter case is 13.624.160 ptas., which is within less than 1% of the previous case, and was obtained in 5 minutes and 9 seconds. The differences observed in the inventory distribution between both cases clearly indicate that production shifting occurs, essentially in the case of product C. Figure 13 indicates the redistribution of equipment operation times due to steam limitations in the short term, while Figure 14 shows the overall occupation of the plant along the 12 moth periods. The associated inventory redestribution and production evolution to meet the market demand appears in Figures 15 and 16 respectively.

7. FINAL CONSIDERATIONS

Production planning in multipurpose plants is indeed a difficult task to perform and the mathematical formulation of the overall problem is a complex optimization problem with too many variables and restrictions, when applied to real plant situations, to consider the use of conventional optimization solvers. In this sense our solution approach has attempted to identify the critical bottlenecks in the computation procedure and develop appropriate heuristic rules to help simplify the dimensionality of the problem in a rational way. Furthermore we believe that it is the right time to develop practical computational tools for the practicing engineer for the kind of work most often left to his intuition and improvisation.

The software package presented here, integrated in today's modestly-priced workstations and provided with easy-to-use interfaces, should be the answer to present-day production problems under increasing economical pressures and which require prompt and accurate decisions.

8. ACKNOWLEDGEMENTS

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9. NOMECLATURE

a = exponetial constant in the operating-time calculation
a_{ijl} = exponential constant for product i in task j and equipment type l in residence-time calculation
A = $m \times n_v$ matrix with rational elements for the zero-one programming problem
$\underline{b}$ = vector of length m with rational elements for the zero-one programming problem
C_{im} = unit cost for product i and dominant production line m (ptas/kg)
$\underline{c}$ = cost vector of length n_v with rational elements for the zero-one programming problem (ptas/kg)
B_{ijn} = batch-production capacity of product i in equipment n used for task j
C_p = amount of product (kg)
D_{it} = demand of product i in period t
F_p = process flow (kg/s)
G_{kijn} = demand of utility k at stage j for equipment type n
G_{okijl} = independent constant in utilities-demand computation
G_{lkijl} = linear constant in the utilities-demand computation
h_i = inventory cost for product i and unit production period
I_{it} = inventory of product i in period t
K_j = equipment set used for task j
m = number of restricitons in the zero-one programming problem
n_b = number of batch types
n_e = number of available equipment
n_j = number of tasks
n_p = number of products

n_t = number of periods

n_v = zero-one variable number for the integer programming problem

n_w = number of tasks to be sequenced in the time interval specified

N_{imt} = number of batches ofproduct i to be produced in the period t following processing line m

P_1 = production-line parameter defined as: Process capacity/total Processing Time (kg/min)

P_2 = equipment idleness level

P_3 = production line parameter defined as: Overall production Cost/Process capacity (ptas/kg)

P_4 = production-line parameter = $P_1 / P_2 . P_3$ (ptas/kg)

R_{ijn} = maximum production capacity of product i for task j in equipment n

S_{ijn} = capacity factor for equipment n, product i and task j

t = time period (s)

T_{ijn} = residence time for product i and task j and class of equipment n (s)

T_{oij1} = independent constant for residence-time calculation

T_{1ij1} = independent constant for residence-time calculation

T_p = total processing time (s)

T_r = residence time (s)

T_0 = linear constant in residence-time calculation (s)

T_1 = production-capacity dependent coefficient in the residence-time calculation ($\mathrm{s/kg}^{-a}$)

X_{im} = production capacity of line m for the product i

X_i = zero-one variable for the integer programming problem, $i = 1, ..., n_v$

$\underline{X}$ = zero-one variables vector.

SUBSCRIPTS

i = product number

j = task number

k = utility type

m = production-line number

n = equipment type number

t = period number

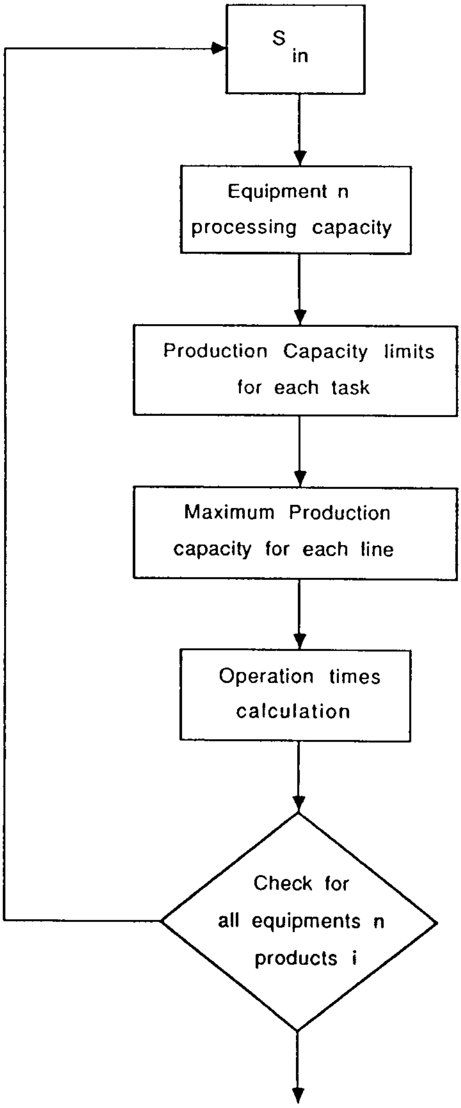

FIGURA 1

Calculation procedure to determine the maximum production capacity and processing time for each task.

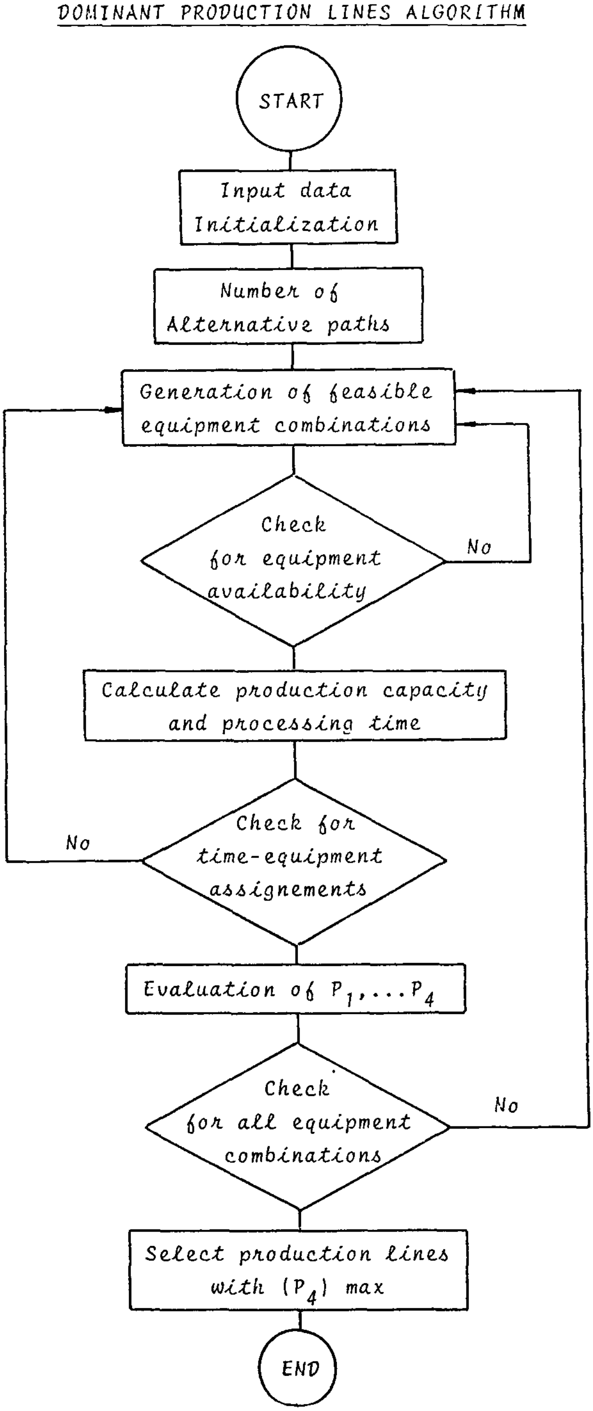

FIGURA 2

Flowchart of the algorithm wich obtains the dominant processing lines

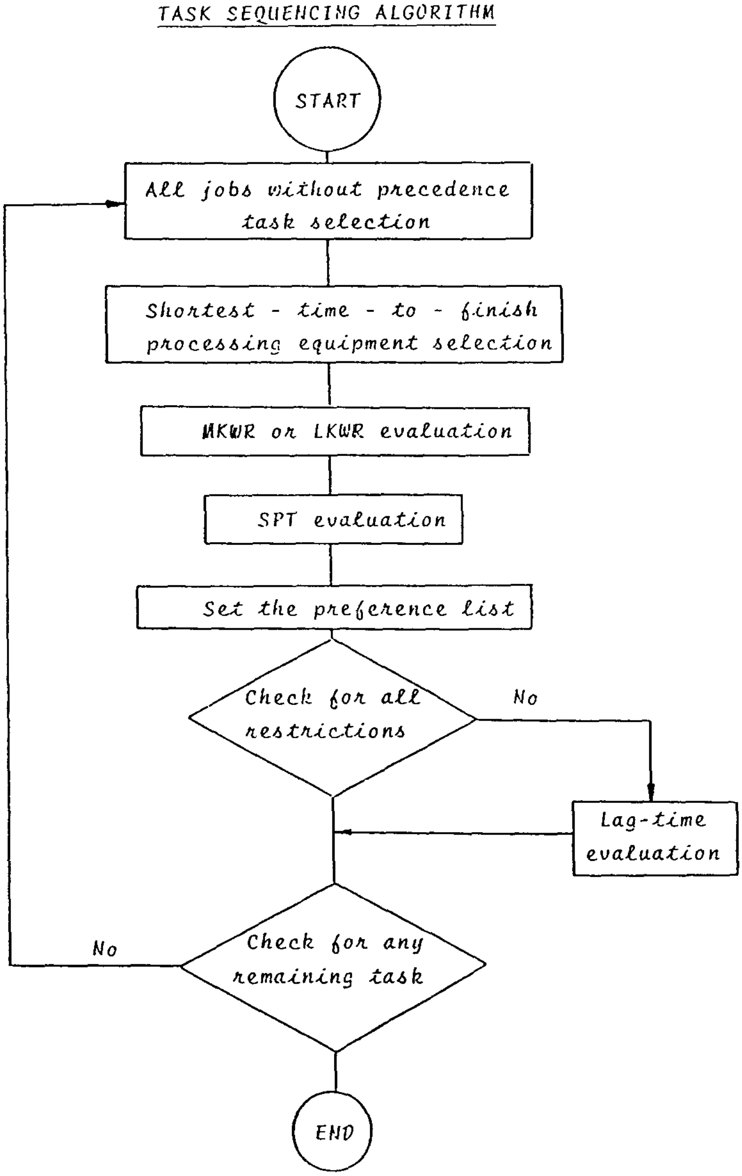

FIGURA 3

Task - sequencing algorithm

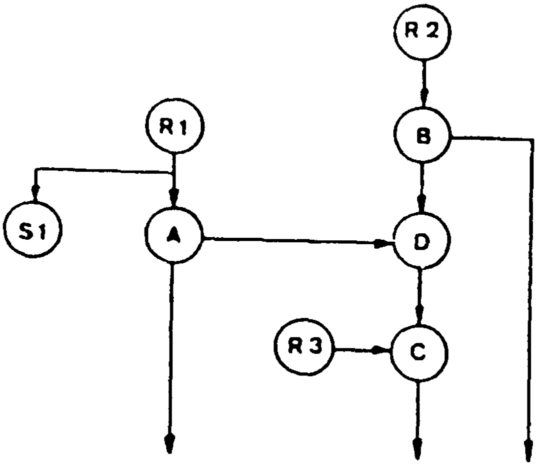

FIGURA 4

Product categories and diagram of possible interdependencies. C is the final product and S1 is the byproduct, while R1, R2, R3 are the raw materials. Intermediate products are A, B (with specified demand) and D required to produce C.

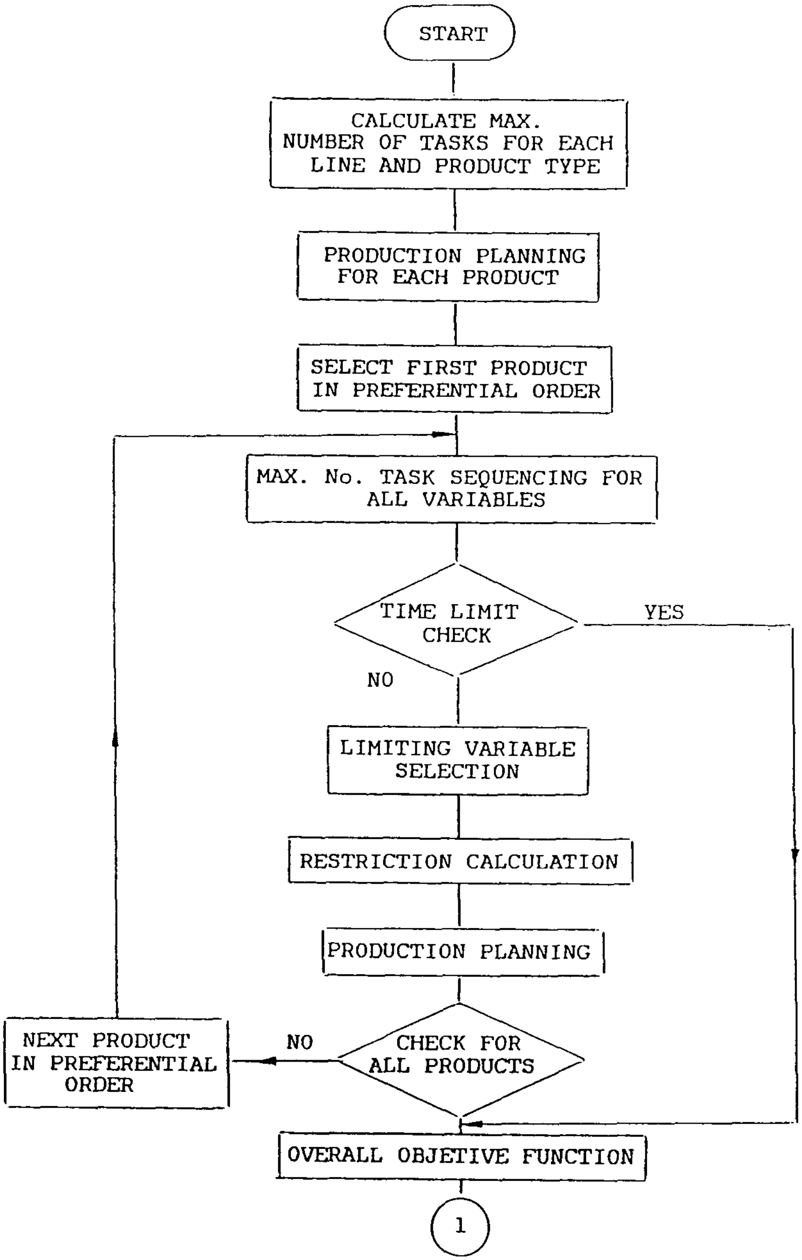

FIGURA 5a

Simplified Flowchart of the global algorithm for the optimal production planning (part one).

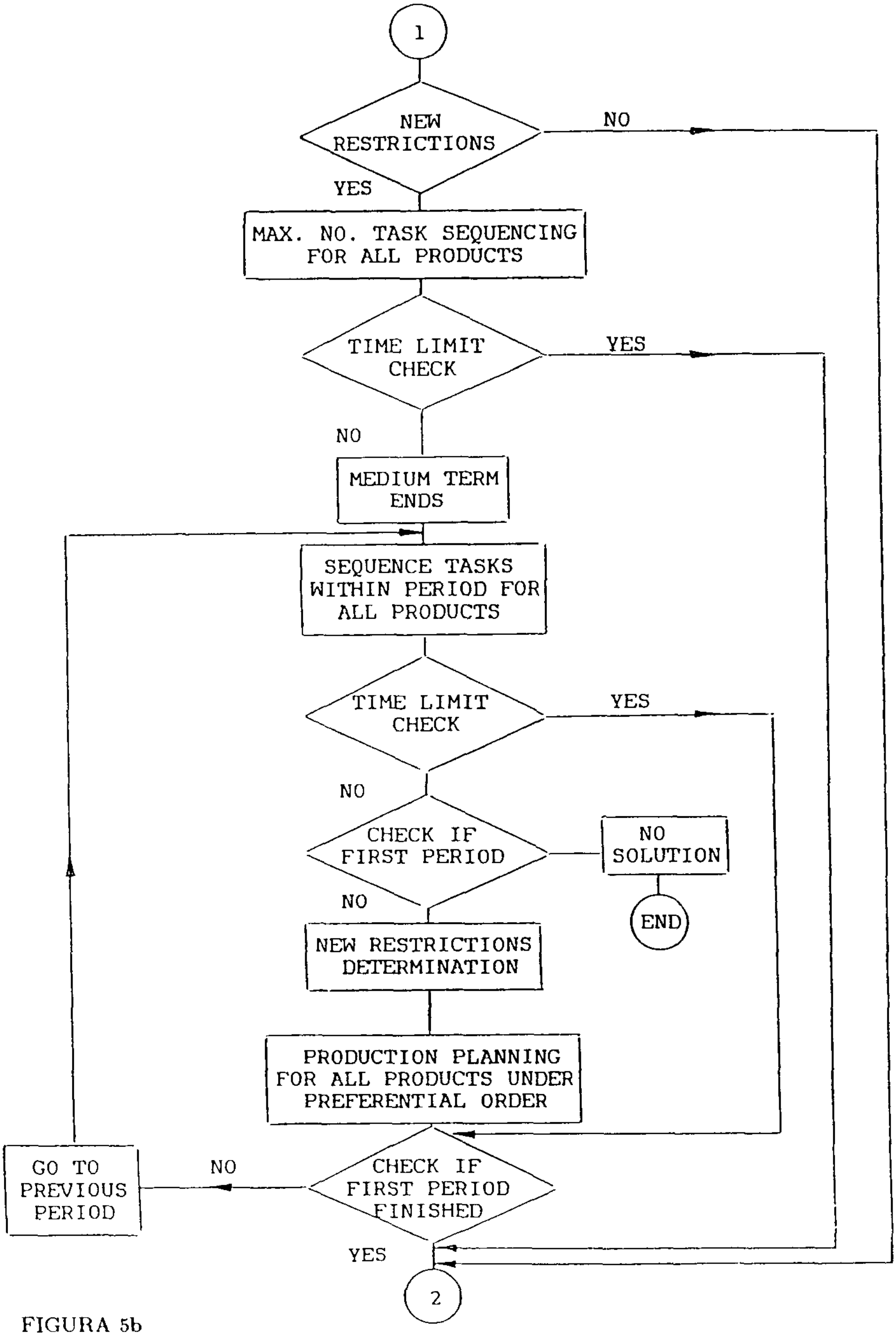

FIGURA 5b

Simplified flowchart of the global algorithm for the optimal production planning under restrictions (part two).

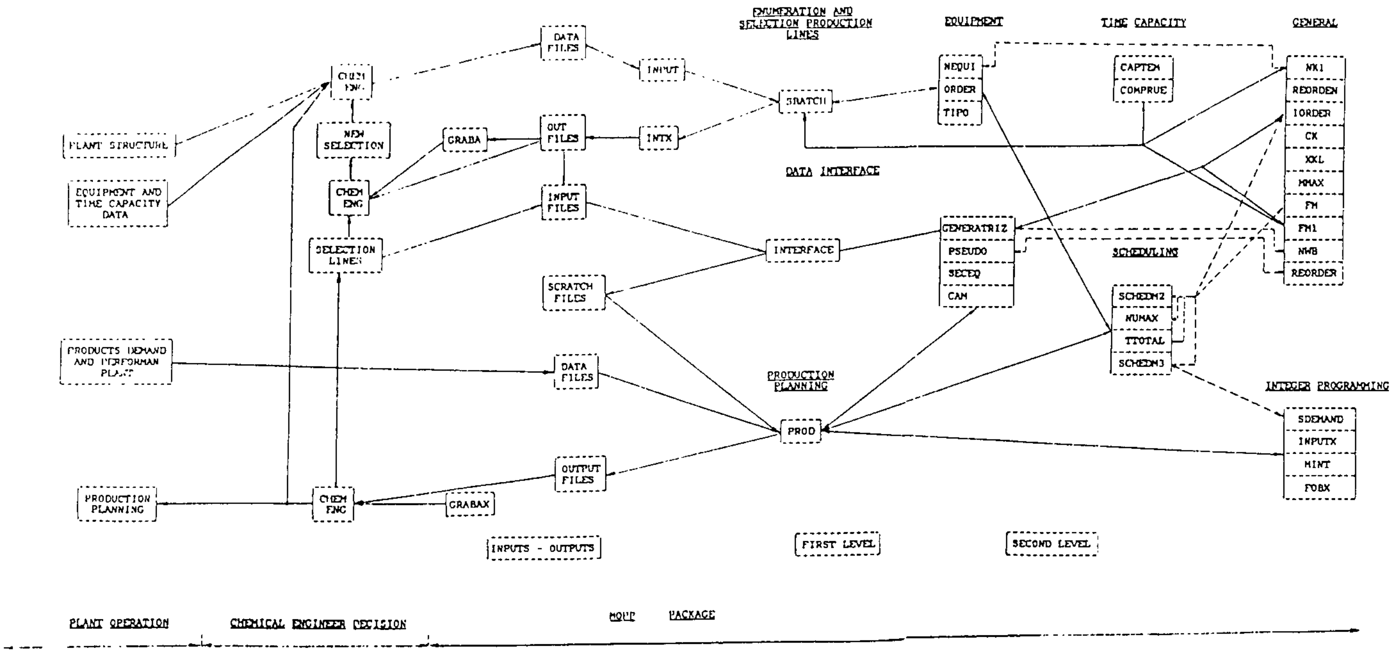

FIGURA 6

Program structure.

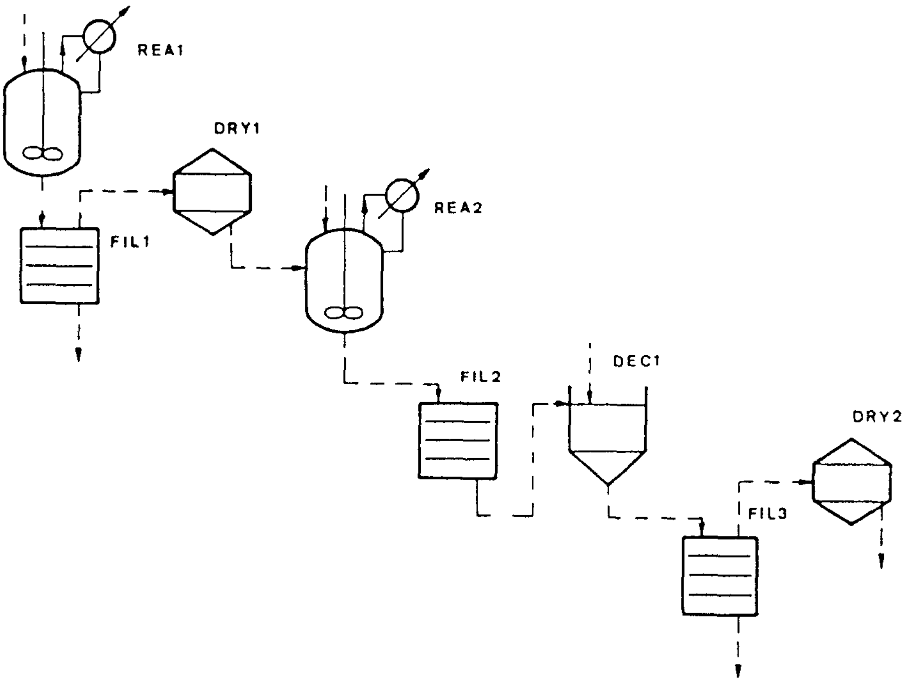

FIGURA 7

Schematic representation of the manufacturing process for product A. Continous and discontinous lines indicate semicontinous and batch operation respectively.

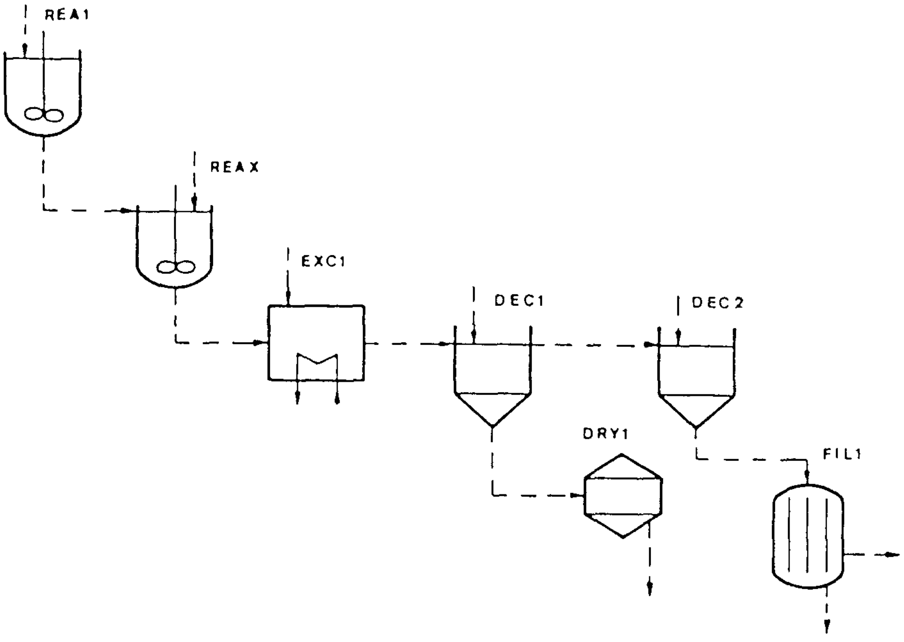

FIGURA 8

Schematic representation of the manufacturing process for product B. (see Figure 7 for comments).

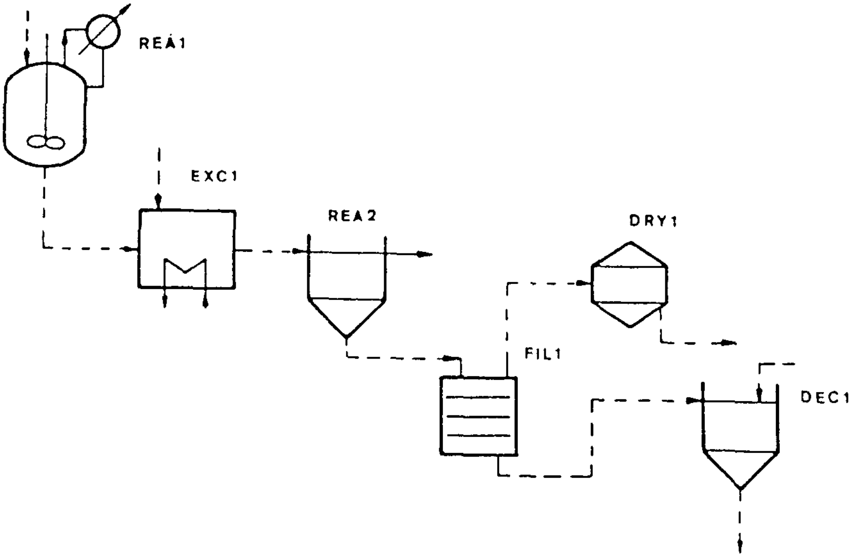

FIGURA 9

Schematic representation of the manufacturing process for product C. (See Figure 7 for comments).

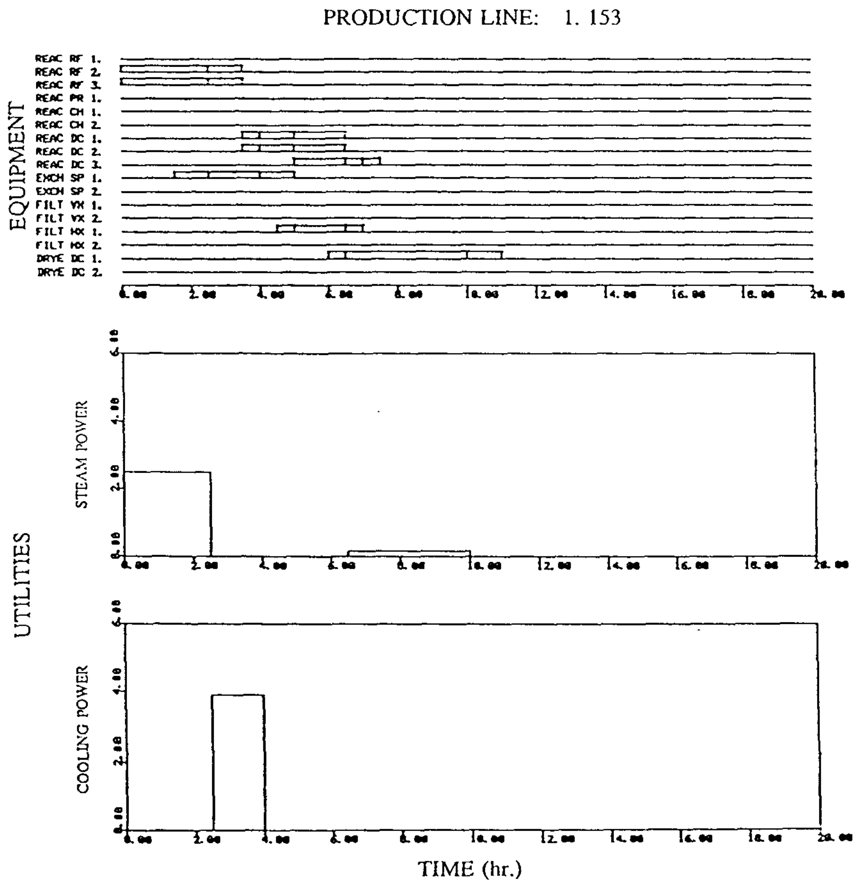

FIGURA 10

Gantt chart for the production line 1.153 for product A showing the equipment operation times and utilities required.

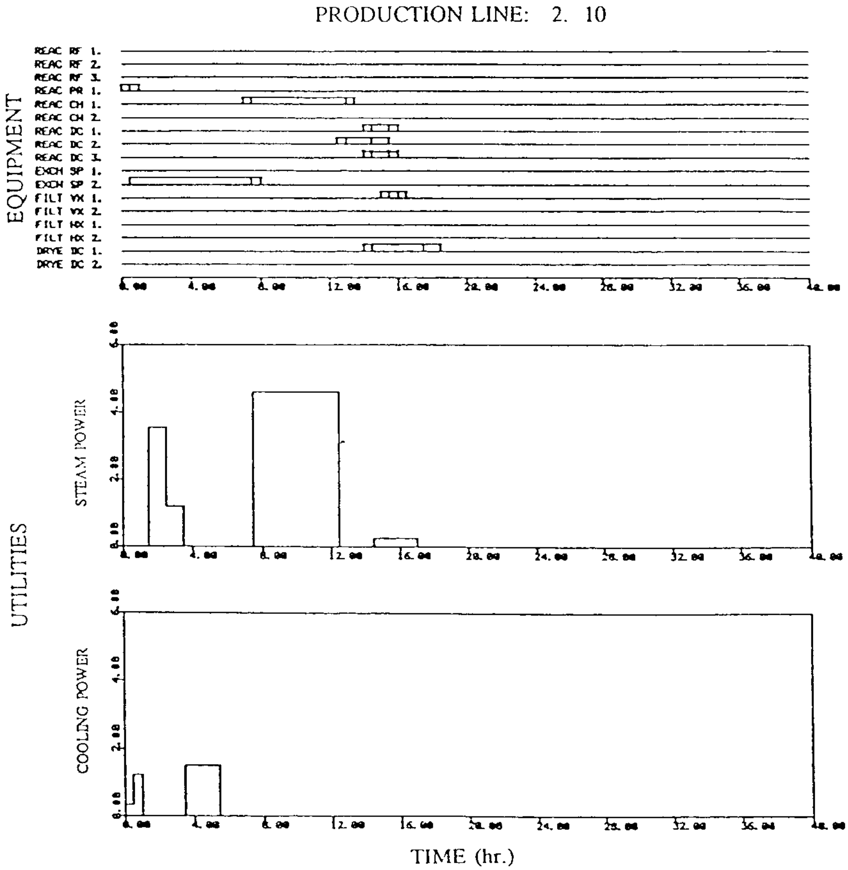

FIGURA 11

Gantt chart for the production line 2.10 for product B showing the equipment operation times and utilities required.

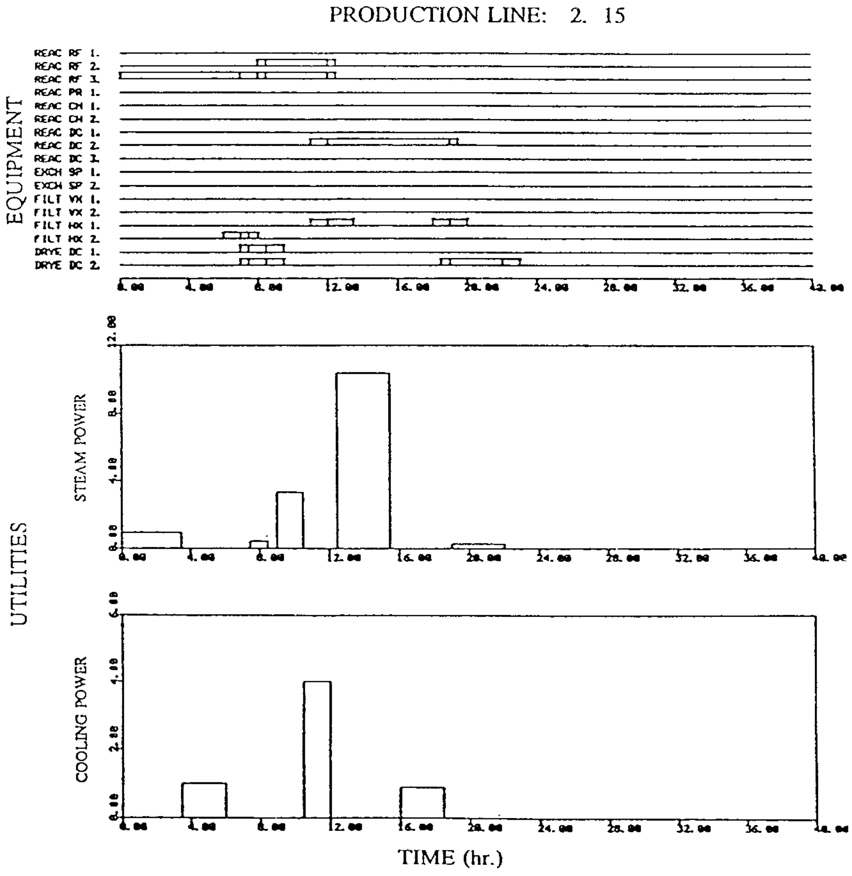

FIGURA 12

Gantt chart for the production line 2.15 for product C showing the equipment operation times and utilities required.

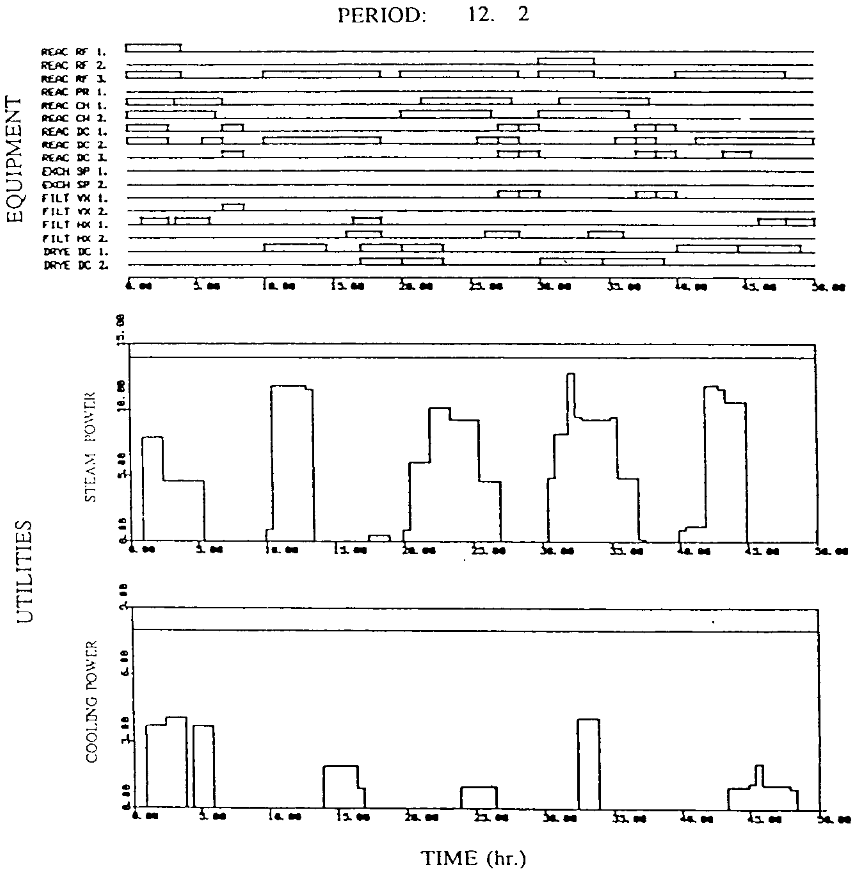

FIGURA 13

Actual redestribution of the equipment operation times in the short term when the plant is operating under utilities limitations.

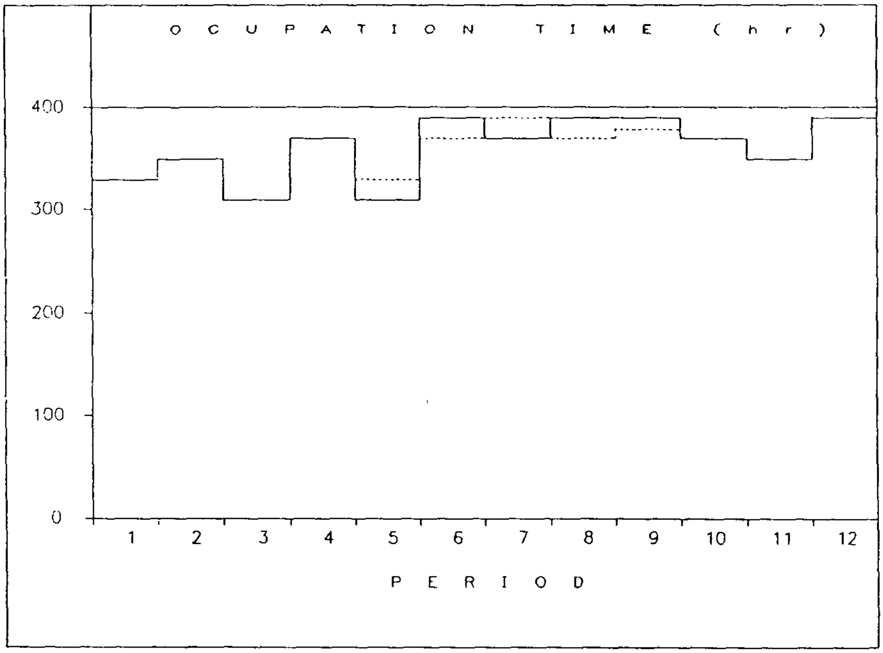

FIGURA 14

Overall occupation of the plant through the 12 month period. The solid line indicates the non-restrictions case, while the broken line shows the plant operation under utility limitations.

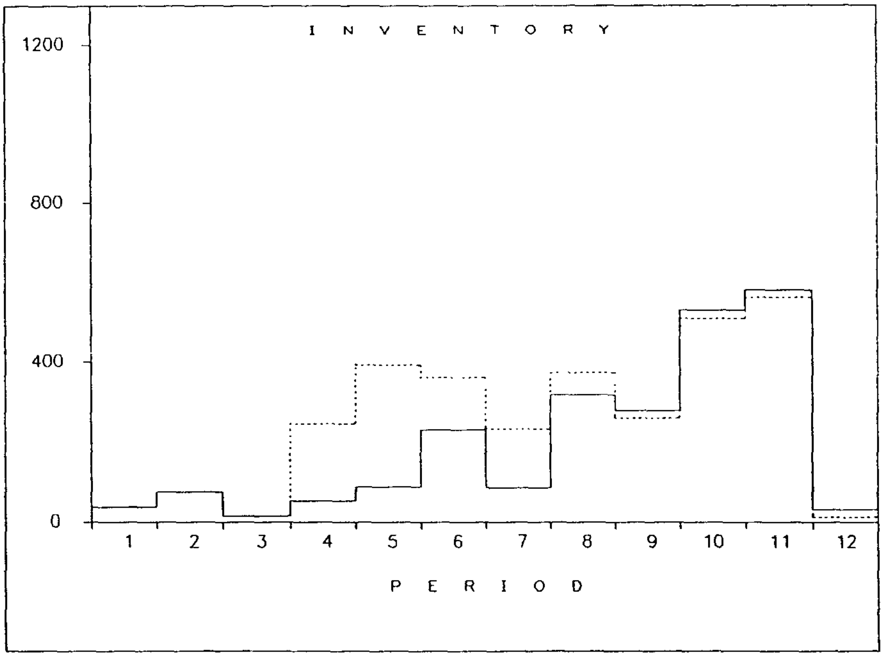

FIGURA 15

Inventory level through the 12 month period. Broken and solid lines correspond to operation under utility limitations and without such limitations respectively.

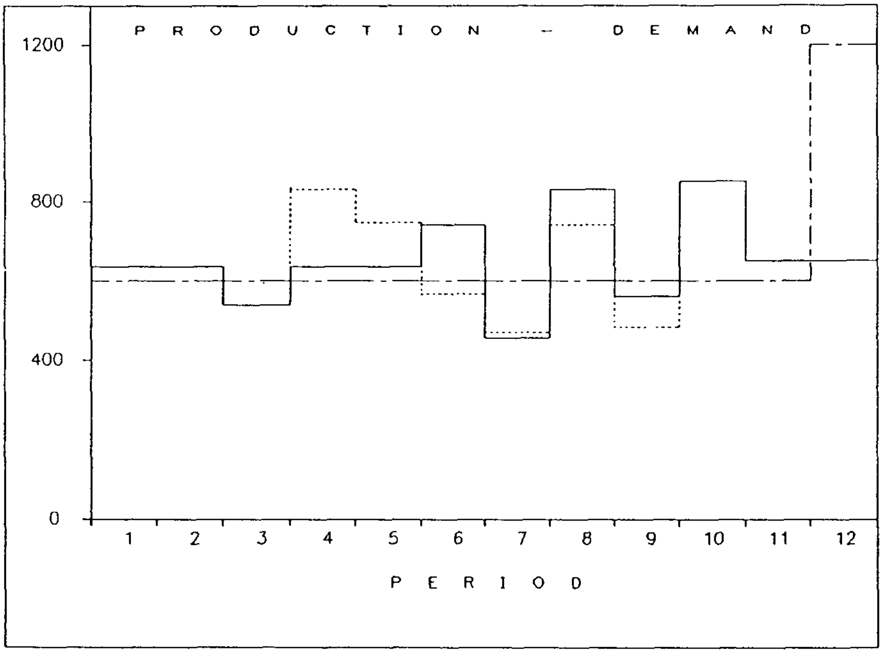

FIGURA 16

Demand and production levels for the 12 monthly periods. Discontinous line (-.-) indicates market demand, while broken and solid lines correspond to the operation in the presence or absence of utility limitations.

TABLE I

Characterization of Equipment Commonly available in Batch Plants

Equipment		Capacity	
Class	Characteristic	Variable	, units
Mixing	Tank	Volume	, m^3
Filtering	Press	Area	, m^2
	Horizontal	Area	, m^2
Drying	Rotary	Area	, m^2
	Fluidized bed	Area	, m^2
	Agitated pans	area	, m^2
Pumping	Pump	Flow	, kg/s
	Compressor	Flow	, kg/s
Heat Exchanger	Tube-type	Area	, m^2
	Evaporator	Area	, m^2
Reactor	Tubular	Volume	, m^3
	CSTR	Volume	, m^3

TABLE II

Values of the Experimental Constant for Residence Time calculation

Equipment	Operation mode	a_{ijl}
Filter	batch(*)	2
Dryer	batch	1
Heater/Cooler	batch	0.5 – 0.66
Reactor	batch	1
Filter	continous	−2
Heat exchanger	continous	−1
CSTR	continous	−1

(*)operating at constant pressure

TABLE III

Indicative Computing Times of the Different Algorithms

Algorithm	problem Dimension	CPU time* min:s)
Task Sequencing	30 Jobs 15 Tasks	0 : 30 – 1 : 50
Dominant Lines	Each Product 15Tasks	0 : 30 – 1 : 00
Production Planning	Each Product 12 Periods 6 Dominant lines	0 : 05 – 0 : 1
Global Algorithm	3 Products 12 Periods 6 Dominant lines	15 : 00 – 20 : 00

* Digital Vax 780 Computer

TABLE IV

Available Equipment

Class	Type	Quantity	Capacity	U	C_u	C_c
REAC	RF	3	0.8	0.50	30.0	10.0
			0.8	0.50	30.0	10.0
			0.4	0.50	20.0	10.0
REAC	PR	1	0.2	0.50	30.0	10.0
REAC	CH	2	0.8	0.50	30.0	10.0
			0.8	0.50	30.0	10.0
REAC	DC	3	0.6	0.50	25.0	10.0
			1.0	0.50	40.0	10.0
			1.5	0.50	50.0	10.0
EXCH	SP	2	1.0	0.50	40.0	10.0
			0.8	0.50	30.0	10.0
FILT	VX	2	6.3	0.50	60.0	15.0
			6.3	0.50	60.0	15.0
FILT	HX	2	3.0	0.50	60.0	15.0
			3.0	0.50	60.0	15.0
DRYE	DC	2	3.0	0.50	60.0	15.0
			3.0	0.50	60.0	15.0

U = Equipment utilization factor ;

C_u = Equipment utilization cost per time unit;

C_c = Equipment cleaning cost per time unit

TABLE VIII

Demand pattern for each product and period

PERIOD

Product	1	2	3	4	5	6	7	8	9	10	11	12	13
A	400	720	720	320	200	800	760	320	600	240	800	600	600
B	300	200	300	260	300	600	800	800	540	160	300	250	250
C	600	600	600	600	600	600	600	600	600	600	600	600	600

TABLE IX

Selected production Lines for Product A

Type	Numbers	Capacity	P_1	P_2 $(\times 10^2)$	P_3	P_4 $(\times 10^2)$
1	152, 153, 185, 186 200, 201, 215, 216	120.0	0.186	1.555	429.8	2.79
2	6, 7, 24, 25, 42, 43 60, 61, 78, 79, 96 97, 114, 115, 132 133, 170, 171	110.0	0.182	2.722	427.7	1.56
3	8, 9, 26, 27, 44, 45 62, 63, 80, 81, 98 99, 116, 117, 134 135, 172, 173	85.0	0.141	20.258	454.5	1.53

TABLE X

Selected Production Lines for Product B

Type	Numbers	Capacity	P_1	P_2	P_3	P_4 $(\times 10^4)$
1	1, 2, 3, 4, 5, 6, 7, 8 11, 12, 15, 16	100.0	0.091	0.285	583.8	5.45
2	9, 10, 13, 14, 17, 18	100.0	0.091	0.340	626.9	4.27

TABLE XI

Selected production Lines for Product C

Type	Numbers	Capacity	P_1	P_2	P_3	P_4 $(\times 10^4)$
1	33, 34, 37, 38, 39 100, 103, 104, 165 166, 169, 170, 231 232, 235, 236	97.0	0.071	0.207	961.4	3.59
2	3, 4, 7, 8, 11, 12, 15 16, 69, 70, 73, 74 77, 78, 81, 82.135 136, 139, 140, 143 144, 147, 148, 201 202, 205, 206, 209 210, 213, 214 232, 235, 236	90.0	0.062	0.22	943.1	3.19

TABLE XII

Products inventory obtained for unlimited utilities

	PERIOD												
Product	1	2	3	4	5	6	7	8	9	10	11	12	13
A	40	90	30	40	60	30	40	50	–	90	60	670	70
B	–	–	–	40	40	40	40	40	40	40	40	290	40
C	37	74	14	245	289	243	100	234	292	447	588	639	39

TABLE XIII

Products inventory obtained under actual utilities limitations

	PERIOD												
Product	1	2	3	4	5	6	7	8	9	10	11	12	13
A	40	90	30	40	60	30	40	50	–	90	60	670	70
B	–	–	–	40	40	40	40	40	40	40	40	290	40
C	37	74	14	155	296	354	336	297	355	517	568	619	19

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