

**Supplemental material for “Inference on the symmetry point-based optimal cut-off point and associated sensitivity and specificity with application to SARS-CoV-2 antibody data”**

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June 2023

The material contained herein is supplementary to the article named in the title and published in SORT-Statistics and Operations Research Transactions Volume 47(1).

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## Results from the Simulation Study of Section 3.1

**Table 1.** Estimated coverage probabilities of the confidence intervals for  $c_s$ , when  $X \sim N(\mu_X = 6.5, \sigma_X^2 = 0.3^2)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods						
		$\delta$	$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
6.81	(20,30)	0.933	0.915	0.944	0.949	0.937	0.957	0.957
	(30,30)	0.932	0.908	0.935	0.932	0.931	0.946	0.947
	(50,50)	0.938	0.938	0.926	0.922	0.925	0.956	0.957
	(50,100)	0.925	0.928	0.940	0.942	0.939	0.953	0.953
	(100,100)	0.937	0.913	0.940	0.941	0.930	0.954	0.954
	(50,300)	0.939	0.940	0.945	0.940	0.944	0.952	0.952
6.99	(20,30)	0.931	0.943	0.924	0.924	0.922	0.956	0.956
	(30,30)	0.951	0.941	0.955	0.953	0.949	0.952	0.952
	(50,50)	0.934	0.946	0.939	0.932	0.931	0.968	0.968
	(50,100)	0.936	0.931	0.936	0.928	0.931	0.954	0.954
	(100,100)	0.933	0.937	0.939	0.944	0.929	0.953	0.953
	(50,300)	0.935	0.933	0.934	0.929	0.927	0.938	0.938
7.25	(20,30)	0.928	0.936	0.932	0.928	0.927	0.951	0.951
	(30,30)	0.938	0.946	0.935	0.930	0.924	0.967	0.967
	(50,50)	0.943	0.948	0.946	0.943	0.946	0.960	0.960
	(50,100)	0.955	0.922	0.955	0.950	0.949	0.951	0.951
	(100,100)	0.954	0.941	0.947	0.943	0.943	0.958	0.958
	(50,300)	0.945	0.942	0.934	0.928	0.933	0.960	0.960

**Table 2.** Estimated coverage probabilities of the confidence intervals for  $c_s$ , when  $X \sim N(\mu_X = 3.5, \sigma_X^2 = 0.3^2)^{-3}$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)^{-3}$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
3.19	(20,30)	0.924	0.936	0.940	0.930	0.959	0.952
	(30,30)	0.931	0.929	0.923	0.927	0.946	0.953
	(50,50)	0.938	0.933	0.927	0.914	0.941	0.937
	(50,100)	0.920	0.945	0.940	0.935	0.961	0.943
	(100,100)	0.912	0.928	0.928	0.921	0.950	0.953
	(50,300)	0.945	0.946	0.943	0.939	0.953	0.953
3.01	(20,30)	0.927	0.923	0.910	0.911	0.961	0.958
	(30,30)	0.948	0.957	0.950	0.951	0.955	0.965
	(50,50)	0.933	0.947	0.943	0.937	0.958	0.956
	(50,100)	0.948	0.944	0.947	0.934	0.948	0.965
	(100,100)	0.918	0.932	0.925	0.929	0.962	0.968
	(50,300)	0.943	0.941	0.938	0.936	0.963	0.963
2.75	(20,30)	0.931	0.935	0.930	0.933	0.961	0.947
	(30,30)	0.941	0.939	0.933	0.936	0.950	0.960
	(50,50)	0.933	0.949	0.936	0.939	0.959	0.960
	(50,100)	0.952	0.953	0.952	0.955	0.956	0.950
	(100,100)	0.942	0.942	0.938	0.938	0.966	0.948
	(50,300)	0.938	0.929	0.928	0.927	0.945	0.945

**Table 3.** Estimated coverage probabilities of the confidence intervals for  $c_s$ , when  $\log(X) \sim N(\mu_X = 2.5, \sigma_X^2 = 0.3^2)$  and  $\log(Y) \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
2.81	(20,30)	0.926	0.943	0.950	0.936	0.956	0.951
	(30,30)	0.931	0.928	0.931	0.933	0.953	0.955
	(50,50)	0.940	0.922	0.923	0.924	0.952	0.939
	(50,100)	0.914	0.936	0.939	0.935	0.953	0.949
	(100,100)	0.928	0.944	0.941	0.929	0.951	0.956
	(50,300)	0.941	0.945	0.939	0.946	0.952	0.952
2.99	(20,30)	0.926	0.927	0.923	0.923	0.949	0.955
	(30,30)	0.939	0.955	0.954	0.947	0.952	0.958
	(50,50)	0.925	0.940	0.931	0.931	0.949	0.959
	(50,100)	0.937	0.937	0.928	0.930	0.949	0.956
	(100,100)	0.929	0.939	0.942	0.930	0.953	0.952
	(50,300)	0.933	0.938	0.930	0.930	0.939	0.939
3.25	(20,30)	0.928	0.929	0.928	0.925	0.956	0.950
	(30,30)	0.937	0.935	0.930	0.924	0.951	0.953
	(50,50)	0.936	0.945	0.943	0.943	0.962	0.961
	(50,100)	0.951	0.956	0.951	0.949	0.962	0.961
	(100,100)	0.953	0.945	0.942	0.944	0.955	0.951
	(50,300)	0.943	0.936	0.928	0.933	0.957	0.957

**Table 4.** Estimated coverage probabilities of the confidence intervals for  $c_s$ , when  $X \sim \text{Gamma}(\alpha_X = 2, \beta_X = 2)$  and  $Y \sim \text{Gamma}(\alpha_Y = 2, \beta_Y)$ .

Scenario $\beta_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
3.51	(20,30)	0.915	0.940	0.947	0.943	0.951	0.951
	(30,30)	0.920	0.924	0.923	0.924	0.947	0.947
	(50,50)	0.929	0.920	0.920	0.910	0.948	0.948
	(50,100)	0.916	0.939	0.935	0.938	0.950	0.950
	(100,100)	0.933	0.944	0.948	0.947	0.942	0.942
	(50,300)	0.934	0.936	0.939	0.942	0.953	0.953
4.97	(20,30)	0.937	0.919	0.915	0.920	0.956	0.956
	(30,30)	0.912	0.936	0.935	0.935	0.944	0.944
	(50,50)	0.939	0.946	0.948	0.944	0.965	0.965
	(50,100)	0.945	0.937	0.934	0.935	0.966	0.966
	(100,100)	0.941	0.948	0.948	0.950	0.965	0.965
	(50,300)	0.946	0.944	0.940	0.942	0.951	0.951
8.23	(20,30)	0.936	0.940	0.938	0.937	0.951	0.951
	(30,30)	0.944	0.931	0.934	0.927	0.948	0.948
	(50,50)	0.938	0.950	0.950	0.950	0.955	0.955
	(50,100)	0.933	0.939	0.939	0.945	0.959	0.959
	(100,100)	0.947	0.946	0.942	0.943	0.969	0.969
	(50,300)	0.950	0.947	0.940	0.944	0.939	0.939

**Table 5.** Estimated coverage probabilities of the confidence intervals for  $c_s$ , when  $X \sim 0.5N(10, 1) + 0.5N(13, 1)$  and  $Y \sim 0.5N(\mu_{Y_1}, 1) + 0.5N(\mu_{Y_2}, \sqrt{5})$ .

Scenario ( $\mu_{Y_1}, \mu_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(11.43, 15.43)	(20, 30)	0.909	0.925	0.938	0.943	0.953	0.953
	(30, 30)	0.953	0.936	0.937	0.935	0.943	0.943
	(50, 50)	0.980	0.935	0.932	0.919	0.952	0.952
	(50, 100)	0.900	0.918	0.929	0.932	0.945	0.945
	(100, 100)	0.951	0.935	0.941	0.932	0.953	0.953
	(50, 300)	0.937	0.929	0.924	0.929	0.953	0.953
(12.39, 16.39)	(20, 30)	0.978	0.935	0.934	0.924	0.952	0.952
	(30, 30)	0.912	0.937	0.939	0.934	0.964	0.964
	(50, 50)	0.947	0.935	0.936	0.936	0.956	0.956
	(50, 100)	0.972	0.912	0.912	0.917	0.952	0.952
	(100, 100)	0.937	0.950	0.948	0.953	0.954	0.954
	(50, 300)	0.948	0.910	0.896	0.910	0.949	0.949
(13.51, 17.51)	(20, 30)	0.949	0.930	0.933	0.935	0.968	0.968
	(30, 30)	0.982	0.942	0.937	0.936	0.966	0.966
	(50, 50)	0.925	0.954	0.954	0.953	0.957	0.957
	(50, 100)	0.936	0.922	0.918	0.920	0.967	0.967
	(100, 100)	0.969	0.919	0.918	0.912	0.968	0.968
	(50, 300)	0.978	0.927	0.911	0.922	0.959	0.959

**Table 6.** Estimated coverage probabilities of the confidence intervals for  $c_s$ , when  $X \sim 0.75Beta(1, 3) + 0.25Beta(5, 1.75)$  and  $Y \sim 0.75Beta(\alpha_{Y_1}, 2) + 0.25Beta(\alpha_{Y_2}, 4.5)$ .

Scenario ( $\alpha_{Y_1}, \alpha_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(3.70, 2.00)	(20, 30)	0.866	0.928	0.928	0.940	0.946	0.946
	(30, 30)	0.767	0.875	0.871	0.875	0.949	0.949
	(50, 50)	0.792	0.903	0.900	0.898	0.936	0.936
	(50, 100)	0.817	0.888	0.893	0.898	0.941	0.941
	(100, 100)	0.687	0.814	0.819	0.813	0.952	0.952
	(50, 300)	0.860	0.942	0.936	0.938	0.937	0.937
(7.25, 3.00)	(20, 30)	0.771	0.876	0.885	0.878	0.946	0.946
	(30, 30)	0.732	0.839	0.837	0.842	0.956	0.956
	(50, 50)	0.575	0.740	0.745	0.727	0.945	0.945
	(50, 100)	0.715	0.839	0.853	0.843	0.960	0.960
	(100, 100)	0.783	0.879	0.877	0.885	0.954	0.954
	(50, 300)	0.763	0.909	0.887	0.899	0.952	0.952
(16.50, 7.00)	(20, 30)	0.598	0.757	0.747	0.741	0.952	0.952
	(30, 30)	0.649	0.790	0.767	0.758	0.954	0.954
	(50, 50)	0.631	0.760	0.764	0.763	0.963	0.963
	(50, 100)	0.349	0.514	0.520	0.495	0.953	0.953
	(100, 100)	0.521	0.684	0.695	0.660	0.952	0.952
	(50, 300)	0.710	0.882	0.839	0.851	0.942	0.942

**Table 7.** Estimated average lengths of the confidence intervals for  $c_s$ , when  $X \sim N(\mu_X = 6.5, \sigma_X^2 = 0.3^2)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods						
		$\delta$	$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
6.81	(20,30)	0.211	0.209	0.227	0.226	0.225	0.268	0.268
	(30,30)	0.222	0.221	0.231	0.229	0.229	0.279	0.279
	(50,50)	0.243	0.244	0.245	0.243	0.243	0.302	0.302
	(50,100)	0.192	0.189	0.207	0.206	0.206	0.239	0.239
	(100,100)	0.202	0.199	0.212	0.210	0.210	0.250	0.250
	(50,300)	0.115	0.115	0.118	0.117	0.117	0.143	0.143
6.81	(20,30)	0.222	0.220	0.224	0.223	0.223	0.272	0.272
	(30,30)	0.150	0.149	0.161	0.160	0.160	0.186	0.186
	(50,50)	0.157	0.157	0.163	0.162	0.162	0.197	0.197
	(50,100)	0.174	0.173	0.174	0.172	0.172	0.213	0.213
	(100,100)	0.130	0.130	0.136	0.135	0.135	0.162	0.162
	(50,300)	0.121	0.121	0.122	0.121	0.121	0.150	0.150
6.81	(20,30)	0.137	0.136	0.140	0.139	0.139	0.170	0.170
	(30,30)	0.151	0.150	0.150	0.149	0.149	0.186	0.186
	(50,50)	0.107	0.106	0.114	0.113	0.113	0.134	0.134
	(50,100)	0.112	0.112	0.116	0.115	0.115	0.140	0.140
	(100,100)	0.123	0.124	0.123	0.122	0.123	0.152	0.152
	(50,300)	0.133	0.133	0.132	0.131	0.130	0.163	0.163

**Table 8.** Estimated average lengths of the confidence intervals for  $c_s$ , when  $X^{-1/3} \sim N(\mu_X = 3.5, \sigma_X^2 = 0.3^2)$  and  $Y^{-1/3} \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)^{-3}$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
3.19	(20,30)	0.005	0.005	0.005	0.005	0.006	0.006
	(30,30)	0.006	0.006	0.006	0.006	0.007	0.007
	(50,50)	0.007	0.007	0.007	0.007	0.009	0.009
	(50,100)	0.004	0.005	0.005	0.005	0.006	0.006
	(100,100)	0.005	0.005	0.005	0.005	0.006	0.006
	(50,300)	0.003	0.003	0.003	0.003	0.003	0.003
3.01	(20,30)	0.006	0.006	0.006	0.006	0.008	0.008
	(30,30)	0.003	0.004	0.004	0.004	0.004	0.004
	(50,50)	0.004	0.004	0.004	0.004	0.005	0.005
	(50,100)	0.005	0.005	0.005	0.005	0.006	0.006
	(100,100)	0.003	0.003	0.003	0.003	0.004	0.004
	(50,300)	0.003	0.003	0.003	0.003	0.004	0.004
2.75	(20,30)	0.003	0.003	0.003	0.003	0.004	0.004
	(30,30)	0.004	0.004	0.004	0.004	0.005	0.005
	(50,50)	0.002	0.003	0.003	0.003	0.003	0.003
	(50,100)	0.003	0.003	0.003	0.003	0.003	0.003
	(100,100)	0.003	0.003	0.003	0.003	0.004	0.004
	(50,300)	0.004	0.004	0.004	0.004	0.005	0.005

**Table 9.** Estimated average lengths of the confidence intervals for  $c_s$ , when  $\log(X) \sim N(\mu_X = 2.5, \sigma_X^2 = 0.3^2)$  and  $\log(Y) \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
2.81	(20,30)	2.875	3.129	3.108	3.105	3.677	3.685
	(30,30)	3.215	3.381	3.349	3.360	4.132	4.107
	(50,50)	3.909	3.970	3.926	3.941	4.937	4.953
	(50,100)	2.608	2.853	2.826	2.828	3.286	3.280
	(100,100)	2.956	3.131	3.096	3.095	3.709	3.664
	(50,300)	1.571	1.611	1.600	1.599	1.951	1.951
2.99	(20,30)	3.588	3.662	3.633	3.627	4.415	4.480
	(30,30)	2.042	2.206	2.194	2.184	2.549	2.608
	(50,50)	2.302	2.402	2.386	2.386	2.874	2.873
	(50,100)	2.811	2.837	2.806	2.806	3.501	3.467
	(100,100)	1.769	1.860	1.847	1.846	2.222	2.236
	(50,300)	1.771	1.785	1.766	1.769	2.201	2.201
3.25	(20,30)	1.995	2.044	2.026	2.034	2.498	2.476
	(30,30)	2.426	2.424	2.397	2.404	2.995	3.033
	(50,50)	1.462	1.558	1.543	1.545	1.839	1.825
	(50,100)	1.645	1.698	1.681	1.683	2.046	2.036
	(100,100)	1.992	1.999	1.975	1.983	2.462	2.460
	(50,300)	2.140	2.125	2.107	2.106	2.635	2.635

**Table 10.** Estimated average lengths of the confidence intervals for  $c_s$ , when  $X \sim \text{Gamma}(\alpha_X = 2, \beta_X = 2)$  and  $Y \sim \text{Gamma}(\alpha_Y = 2, \beta_Y)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
3.51	(20,30)	1.834	2.033	2.008	2.022	2.360	2.360
	(30,30)	2.247	2.369	2.344	2.361	2.863	2.863
	(50,50)	3.044	3.052	3.012	3.034	3.736	3.735
	(50,100)	1.650	1.826	1.804	1.815	2.127	2.127
	(100,100)	2.030	2.163	2.136	2.141	2.602	2.601
	(50,300)	0.998	1.037	1.029	1.029	1.259	1.259
4.97	(20,30)	2.779	2.816	2.785	2.788	3.394	3.395
	(30,30)	1.299	1.409	1.398	1.403	1.639	1.639
	(50,50)	1.595	1.675	1.660	1.665	2.013	2.013
	(50,100)	2.187	2.196	2.167	2.173	2.672	2.673
	(100,100)	1.131	1.209	1.199	1.201	1.426	1.426
	(50,300)	1.222	1.243	1.232	1.236	1.535	1.535
8.23	(20,30)	1.383	1.430	1.418	1.417	1.726	1.726
	(30,30)	1.879	1.861	1.843	1.850	2.298	2.298
	(50,50)	0.926	0.994	0.988	0.990	1.169	1.169
	(50,100)	1.142	1.189	1.179	1.178	1.427	1.427
	(100,100)	1.552	1.552	1.538	1.536	1.899	1.899
	(50,300)	1.666	1.636	1.625	1.625	2.049	2.049

**Table 11.** *Estimated average lengths of the confidence intervals for  $c_s$ , when  $X \sim 0.5N(10, 1) + 0.5N(13, 1)$  and  $Y \sim 0.5N(\mu_{Y_1}, 1) + 0.5N(\mu_{Y_2}, \sqrt{5})$ .*

Scenario ( $\mu_{Y_1}, \mu_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(11.43, 15.43)	(20, 30)	1.195	1.337	1.326	1.324	1.546	1.546
	(30, 30)	1.266	1.233	1.218	1.222	1.319	1.319
	(50, 50)	1.381	1.124	1.115	1.117	1.193	1.193
	(50, 100)	1.075	1.216	1.203	1.203	1.391	1.391
	(100, 100)	1.143	1.115	1.104	1.106	1.195	1.195
	(50, 300)	0.647	0.648	0.642	0.642	0.838	0.838
(12.39, 16.39)	(20, 30)	1.253	1.013	0.999	1.005	1.058	1.058
	(30, 30)	0.848	0.953	0.942	0.944	1.102	1.102
	(50, 50)	0.899	0.874	0.866	0.865	0.919	0.919
	(50, 100)	0.981	0.788	0.781	0.781	0.809	0.809
	(100, 100)	0.733	0.795	0.787	0.785	0.955	0.955
	(50, 300)	0.690	0.624	0.619	0.618	0.693	0.693
(13.51, 17.51)	(20, 30)	0.779	0.745	0.739	0.738	0.788	0.788
	(30, 30)	0.852	0.682	0.678	0.678	0.709	0.709
	(50, 50)	0.607	0.675	0.670	0.669	0.788	0.788
	(50, 100)	0.641	0.620	0.615	0.615	0.643	0.643
	(100, 100)	0.698	0.561	0.557	0.557	0.570	0.570
	(50, 300)	0.750	0.595	0.591	0.590	0.616	0.616

**Table 12.** *Estimated average lengths of the confidence intervals for  $c_s$ , when  $X \sim 0.75Beta(1, 3) + 0.25Beta(5, 1.75)$  and  $Y \sim 0.75Beta(\alpha_{Y_1}, 2) + 0.25Beta(\alpha_{Y_2}, 4.5)$ .*

Scenario ( $\alpha_{Y_1}, \alpha_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(3.70, 2.00)	(20, 30)	0.152	0.197	0.194	0.195	0.248	0.248
	(30, 30)	0.154	0.206	0.203	0.204	0.274	0.274
	(50, 50)	0.142	0.187	0.185	0.186	0.243	0.243
	(50, 100)	0.137	0.175	0.172	0.173	0.226	0.226
	(100, 100)	0.141	0.185	0.183	0.183	0.249	0.249
	(50, 300)	0.080	0.101	0.100	0.099	0.139	0.139
(7.25, 3.00)	(20, 30)	0.128	0.169	0.168	0.168	0.225	0.225
	(30, 30)	0.107	0.136	0.134	0.135	0.178	0.178
	(50, 50)	0.111	0.144	0.143	0.143	0.196	0.196
	(50, 100)	0.101	0.131	0.130	0.130	0.180	0.180
	(100, 100)	0.092	0.119	0.117	0.118	0.155	0.155
	(50, 300)	0.077	0.108	0.107	0.106	0.157	0.157
(16.50, 7.00)	(20, 30)	0.093	0.124	0.123	0.123	0.176	0.176
	(30, 30)	0.086	0.108	0.107	0.107	0.161	0.161
	(50, 50)	0.077	0.096	0.096	0.095	0.129	0.129
	(50, 100)	0.079	0.102	0.101	0.101	0.144	0.144
	(100, 100)	0.072	0.094	0.093	0.093	0.132	0.132
	(50, 300)	0.069	0.099	0.098	0.098	0.143	0.143

**Table 13.** Estimated coverage probabilities of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim N(\mu_X = 6.5, \sigma_X^2 = 0.3^2)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods						
		$\delta$	$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.945	0.947	0.965	0.941	0.954	0.958	0.958
	(30, 30)	0.944	0.943	0.954	0.914	0.945	0.965	0.965
	(50, 50)	0.940	0.935	0.938	0.877	0.917	0.961	0.961
	(50, 100)	0.956	0.942	0.957	0.935	0.951	0.955	0.955
	(100, 100)	0.938	0.931	0.945	0.897	0.938	0.959	0.959
	(50, 300)	0.951	0.950	0.951	0.942	0.944	0.957	0.957
(16.50, 7.00)	(20, 30)	0.956	0.955	0.959	0.907	0.944	0.965	0.965
	(30, 30)	0.956	0.957	0.962	0.947	0.958	0.962	0.962
	(50, 50)	0.949	0.947	0.952	0.927	0.949	0.960	0.960
	(50, 100)	0.941	0.939	0.942	0.920	0.937	0.961	0.961
	(100, 100)	0.948	0.944	0.950	0.934	0.955	0.969	0.969
	(50, 300)	0.942	0.939	0.945	0.934	0.940	0.949	0.949
(16.50, 7.00)	(20, 30)	0.959	0.959	0.962	0.944	0.951	0.969	0.969
	(30, 30)	0.958	0.957	0.956	0.931	0.946	0.965	0.965
	(50, 50)	0.955	0.953	0.957	0.950	0.955	0.944	0.944
	(50, 100)	0.946	0.947	0.950	0.939	0.942	0.955	0.955
	(100, 100)	0.942	0.938	0.946	0.930	0.939	0.966	0.966
	(50, 300)	0.950	0.947	0.955	0.929	0.946	0.960	0.960

**Table 14.** Estimated coverage probabilities of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim N(\mu_X = 3.5, \sigma_X^2 = 0.3^2)^{-3}$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)^{-3}$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.948	0.962	0.935	0.951	0.968	0.968
	(30, 30)	0.939	0.945	0.913	0.938	0.960	0.960
	(50, 50)	0.928	0.940	0.873	0.911	0.971	0.971
	(50, 100)	0.949	0.960	0.934	0.958	0.958	0.958
	(100, 100)	0.931	0.938	0.915	0.932	0.973	0.973
	(50, 300)	0.945	0.944	0.941	0.942	0.963	0.963
(16.50, 7.00)	(20, 30)	0.951	0.942	0.907	0.925	0.972	0.972
	(30, 30)	0.948	0.956	0.937	0.950	0.972	0.972
	(50, 50)	0.944	0.951	0.917	0.932	0.966	0.966
	(50, 100)	0.936	0.938	0.911	0.922	0.973	0.973
	(100, 100)	0.938	0.944	0.937	0.936	0.969	0.969
	(50, 300)	0.947	0.943	0.938	0.937	0.963	0.963
(16.50, 7.00)	(20, 30)	0.947	0.952	0.938	0.945	0.960	0.960
	(30, 30)	0.940	0.944	0.921	0.937	0.965	0.965
	(50, 50)	0.950	0.952	0.937	0.946	0.968	0.968
	(50, 100)	0.938	0.941	0.928	0.935	0.962	0.962
	(100, 100)	0.952	0.955	0.938	0.944	0.970	0.970
	(50, 300)	0.937	0.939	0.932	0.929	0.959	0.959



**Table 15.** Estimated coverage probabilities of the confidence intervals for  $\text{sens}(c_s)$ , when  $\log(X) \sim N(\mu_X = 2.5, \sigma_X^2 = 0.3^2)$  and  $\log(Y) \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.946	0.966	0.939	0.953	0.957	0.957
	(30, 30)	0.944	0.954	0.914	0.945	0.965	0.965
	(50, 50)	0.933	0.940	0.877	0.911	0.962	0.962
	(50, 100)	0.942	0.958	0.935	0.952	0.960	0.960
	(100, 100)	0.931	0.945	0.897	0.938	0.967	0.967
	(50, 300)	0.952	0.951	0.941	0.944	0.957	0.957
(16.50, 7.00)	(20, 30)	0.956	0.958	0.906	0.944	0.956	0.956
	(30, 30)	0.957	0.963	0.947	0.957	0.963	0.963
	(50, 50)	0.949	0.952	0.927	0.949	0.962	0.962
	(50, 100)	0.939	0.943	0.919	0.932	0.967	0.967
	(100, 100)	0.944	0.950	0.935	0.953	0.970	0.970
	(50, 300)	0.940	0.945	0.934	0.940	0.950	0.950
(16.50, 7.00)	(20, 30)	0.959	0.963	0.943	0.953	0.963	0.963
	(30, 30)	0.958	0.957	0.930	0.944	0.974	0.974
	(50, 50)	0.954	0.957	0.950	0.955	0.946	0.946
	(50, 100)	0.948	0.950	0.936	0.941	0.961	0.961
	(100, 100)	0.939	0.946	0.933	0.937	0.973	0.973
	(50, 300)	0.948	0.954	0.928	0.946	0.958	0.958

**Table 16.** Estimated coverage probabilities of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim \text{Gamma}(\alpha_X = 2, \beta_X = 2)$  and  $Y \sim \text{Gamma}(\alpha_Y = 2, \beta_Y)$ .

Scenario $\beta_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.939	0.956	0.936	0.946	0.969	0.969
	(30, 30)	0.947	0.954	0.917	0.947	0.964	0.964
	(50, 50)	0.957	0.957	0.880	0.937	0.972	0.972
	(50, 100)	0.945	0.958	0.929	0.951	0.972	0.972
	(100, 100)	0.934	0.943	0.912	0.931	0.956	0.956
	(50, 300)	0.937	0.944	0.929	0.936	0.957	0.957
(16.50, 7.00)	(20, 30)	0.941	0.948	0.898	0.923	0.970	0.970
	(30, 30)	0.939	0.944	0.926	0.935	0.954	0.954
	(50, 50)	0.942	0.950	0.924	0.945	0.961	0.961
	(50, 100)	0.939	0.937	0.898	0.925	0.965	0.965
	(100, 100)	0.942	0.948	0.932	0.939	0.962	0.962
	(50, 300)	0.939	0.940	0.920	0.928	0.962	0.962
(16.50, 7.00)	(20, 30)	0.949	0.953	0.935	0.943	0.954	0.954
	(30, 30)	0.944	0.942	0.915	0.939	0.966	0.966
	(50, 50)	0.951	0.953	0.949	0.952	0.956	0.956
	(50, 100)	0.942	0.940	0.929	0.941	0.951	0.951
	(100, 100)	0.940	0.944	0.922	0.934	0.958	0.958
	(50, 300)	0.931	0.930	0.893	0.922	0.961	0.961

**Table 17.** Estimated coverage probabilities of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim 0.5N(10, 1) + 0.5N(13, 1)$  and  $Y \sim 0.5N(\mu_{Y_1}, 1) + 0.5N(\mu_{Y_2}, \sqrt{5})$ .

Scenario ( $\mu_{Y_1}, \mu_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.723	0.724	0.645	0.710	0.940	0.940
	(30, 30)	0.767	0.730	0.582	0.687	0.948	0.948
	(50, 50)	0.921	0.893	0.752	0.844	0.960	0.960
	(50, 100)	0.650	0.632	0.553	0.621	0.945	0.945
	(100, 100)	0.718	0.652	0.536	0.631	0.943	0.943
	(50, 300)	0.217	0.160	0.150	0.167	0.957	0.957
(16.50, 7.00)	(20, 30)	0.930	0.882	0.776	0.851	0.961	0.961
	(30, 30)	0.462	0.431	0.399	0.435	0.952	0.952
	(50, 50)	0.563	0.480	0.394	0.468	0.969	0.969
	(50, 100)	0.911	0.838	0.740	0.801	0.960	0.960
	(100, 100)	0.356	0.320	0.277	0.314	0.949	0.949
	(50, 300)	0.394	0.242	0.196	0.231	0.959	0.959
(16.50, 7.00)	(20, 30)	0.483	0.383	0.301	0.371	0.960	0.960
	(30, 30)	0.884	0.792	0.701	0.762	0.968	0.968
	(50, 50)	0.173	0.154	0.131	0.151	0.959	0.959
	(50, 100)	0.231	0.150	0.117	0.163	0.964	0.964
	(100, 100)	0.832	0.717	0.643	0.713	0.964	0.964
	(50, 300)	0.883	0.789	0.682	0.739	0.957	0.957

**Table 18.** Estimated coverage probabilities of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim 0.75\text{Beta}(1, 3) + 0.25\text{Beta}(5, 1.75)$  and  $Y \sim 0.75\text{Beta}(\alpha_{Y_1}, 2) + 0.25\text{Beta}(\alpha_{Y_2}, 4.5)$ .

Scenario ( $\alpha_{Y_1}, \alpha_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.925	0.966	0.930	0.947	0.948	0.948
	(30, 30)	0.910	0.957	0.904	0.937	0.954	0.954
	(50, 50)	0.892	0.951	0.794	0.855	0.947	0.947
	(50, 100)	0.923	0.946	0.926	0.937	0.963	0.963
	(100, 100)	0.906	0.943	0.903	0.923	0.964	0.964
	(50, 300)	0.899	0.937	0.927	0.929	0.958	0.958
(16.50, 7.00)	(20, 30)	0.882	0.891	0.796	0.843	0.962	0.962
	(30, 30)	0.917	0.940	0.929	0.930	0.966	0.966
	(50, 50)	0.910	0.950	0.912	0.935	0.970	0.970
	(50, 100)	0.860	0.858	0.770	0.813	0.952	0.952
	(100, 100)	0.925	0.943	0.934	0.935	0.947	0.947
	(50, 300)	0.897	0.946	0.928	0.930	0.957	0.957
(16.50, 7.00)	(20, 30)	0.906	0.953	0.922	0.930	0.945	0.945
	(30, 30)	0.791	0.832	0.727	0.769	0.953	0.953
	(50, 50)	0.933	0.946	0.936	0.941	0.959	0.959
	(50, 100)	0.931	0.954	0.930	0.946	0.955	0.955
	(100, 100)	0.738	0.731	0.655	0.694	0.964	0.964
	(50, 300)	0.666	0.830	0.724	0.750	0.952	0.952

**Table 19.** Estimated average lengths of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim N(\mu_X = 6.5, \sigma_X^2 = 0.3^2)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods						
		$\delta$	$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.210	0.209	0.229	0.224	0.225	0.262	0.262
	(30, 30)	0.195	0.195	0.211	0.202	0.204	0.244	0.244
	(50, 50)	0.165	0.165	0.175	0.160	0.164	0.208	0.208
	(50, 100)	0.197	0.196	0.212	0.208	0.209	0.245	0.245
	(100, 100)	0.183	0.183	0.196	0.189	0.190	0.228	0.228
	(50, 300)	0.096	0.096	0.098	0.097	0.096	0.119	0.119
(16.50, 7.00)	(20, 30)	0.157	0.156	0.164	0.153	0.156	0.196	0.196
	(30, 30)	0.154	0.154	0.162	0.160	0.160	0.191	0.191
	(50, 50)	0.144	0.144	0.150	0.147	0.147	0.177	0.177
	(50, 100)	0.122	0.122	0.125	0.120	0.121	0.153	0.153
	(100, 100)	0.122	0.122	0.127	0.125	0.125	0.152	0.152
	(50, 300)	0.090	0.090	0.091	0.089	0.089	0.111	0.111
(16.50, 7.00)	(20, 30)	0.115	0.115	0.118	0.116	0.116	0.142	0.142
	(30, 30)	0.098	0.098	0.100	0.097	0.097	0.123	0.123
	(50, 50)	0.109	0.109	0.113	0.111	0.111	0.135	0.135
	(50, 100)	0.102	0.102	0.104	0.103	0.103	0.126	0.126
	(100, 100)	0.087	0.087	0.088	0.086	0.086	0.108	0.108
	(50, 300)	0.077	0.077	0.077	0.075	0.075	0.095	0.095

**Table 20.** Estimated average lengths of the confidence intervals for  $\text{sens}(c_s)$ , when  $X^{-1/3} \sim N(\mu_X = 3.5, \sigma_X^2 = 0.3^2)$  and  $Y^{-1/3} \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)^{-3}$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.209	0.226	0.222	0.223	0.258	0.258
	(30, 30)	0.195	0.209	0.201	0.202	0.239	0.239
	(50, 50)	0.164	0.173	0.159	0.162	0.205	0.205
	(50, 100)	0.196	0.210	0.207	0.207	0.242	0.242
	(100, 100)	0.184	0.195	0.188	0.190	0.225	0.225
	(50, 300)	0.096	0.097	0.096	0.096	0.118	0.118
(16.50, 7.00)	(20, 30)	0.156	0.163	0.151	0.154	0.194	0.194
	(30, 30)	0.153	0.160	0.158	0.158	0.188	0.188
	(50, 50)	0.143	0.148	0.145	0.145	0.175	0.175
	(50, 100)	0.122	0.125	0.119	0.120	0.151	0.151
	(100, 100)	0.123	0.126	0.125	0.125	0.151	0.151
	(50, 300)	0.090	0.091	0.090	0.090	0.110	0.110
(16.50, 7.00)	(20, 30)	0.115	0.117	0.115	0.115	0.140	0.140
	(30, 30)	0.098	0.099	0.096	0.097	0.119	0.119
	(50, 50)	0.109	0.112	0.111	0.111	0.134	0.134
	(50, 100)	0.102	0.104	0.103	0.103	0.124	0.124
	(100, 100)	0.087	0.088	0.086	0.086	0.107	0.107
	(50, 300)	0.076	0.077	0.076	0.076	0.093	0.093

**Table 21.** Estimated average lengths of the confidence intervals for  $\text{sens}(c_s)$ , when  $\log(X) \sim N(\mu_X = 2.5, \sigma_X^2 = 0.3^2)$  and  $\log(Y) \sim N(\mu_Y, \sigma_Y^2 = 0.5^2)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.209	0.228	0.224	0.225	0.261	0.261
	(30, 30)	0.195	0.210	0.202	0.203	0.242	0.242
	(50, 50)	0.165	0.174	0.160	0.164	0.204	0.204
	(50, 100)	0.196	0.212	0.208	0.209	0.243	0.243
	(100, 100)	0.183	0.195	0.188	0.190	0.226	0.226
	(50, 300)	0.096	0.098	0.097	0.096	0.118	0.118
(16.50, 7.00)	(20, 30)	0.156	0.164	0.152	0.156	0.194	0.194
	(30, 30)	0.154	0.162	0.159	0.159	0.190	0.190
	(50, 50)	0.144	0.149	0.146	0.146	0.176	0.176
	(50, 100)	0.122	0.125	0.119	0.121	0.151	0.151
	(100, 100)	0.122	0.127	0.125	0.125	0.151	0.151
	(50, 300)	0.090	0.091	0.089	0.089	0.110	0.110
(16.50, 7.00)	(20, 30)	0.115	0.118	0.116	0.116	0.141	0.141
	(30, 30)	0.098	0.100	0.096	0.097	0.121	0.121
	(50, 50)	0.109	0.113	0.111	0.111	0.135	0.135
	(50, 100)	0.102	0.104	0.102	0.103	0.125	0.125
	(100, 100)	0.087	0.088	0.086	0.086	0.107	0.107
	(50, 300)	0.077	0.077	0.075	0.075	0.094	0.094

**Table 22.** Estimated average lengths of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim \text{Gamma}(\alpha_X = 2, \beta_X = 2)$  and  $Y \sim \text{Gamma}(\alpha_Y = 2, \beta_Y)$ .

Scenario $\mu_Y$	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.211	0.228	0.224	0.224	0.262	0.262
	(30, 30)	0.197	0.210	0.202	0.203	0.243	0.243
	(50, 50)	0.166	0.174	0.160	0.164	0.207	0.207
	(50, 100)	0.192	0.204	0.201	0.201	0.239	0.239
	(100, 100)	0.181	0.191	0.185	0.186	0.224	0.224
	(50, 300)	0.107	0.109	0.108	0.107	0.132	0.132
(16.50, 7.00)	(20, 30)	0.156	0.163	0.152	0.155	0.194	0.194
	(30, 30)	0.150	0.155	0.153	0.153	0.186	0.186
	(50, 50)	0.141	0.146	0.143	0.143	0.175	0.175
	(50, 100)	0.121	0.124	0.119	0.120	0.152	0.152
	(100, 100)	0.127	0.130	0.128	0.128	0.156	0.156
	(50, 300)	0.097	0.098	0.097	0.097	0.119	0.119
(16.50, 7.00)	(20, 30)	0.117	0.119	0.117	0.117	0.144	0.144
	(30, 30)	0.099	0.100	0.096	0.097	0.122	0.122
	(50, 50)	0.107	0.108	0.107	0.107	0.132	0.132
	(50, 100)	0.100	0.102	0.101	0.101	0.124	0.124
	(100, 100)	0.087	0.088	0.085	0.086	0.107	0.107
	(50, 300)	0.079	0.079	0.077	0.078	0.097	0.097

**Table 23.** Estimated average lengths of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim 0.5N(10, 1) + 0.5N(13, 1)$  and  $Y \sim 0.5N(\mu_{Y_1}, 1) + 0.5N(\mu_{Y_2}, \sqrt{5})$ .

Scenario ( $\mu_{Y_1}, \mu_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.210	0.212	0.209	0.210	0.278	0.278
	(30, 30)	0.196	0.188	0.180	0.182	0.260	0.260
	(50, 50)	0.173	0.155	0.145	0.146	0.222	0.222
	(50, 100)	0.190	0.189	0.187	0.187	0.248	0.248
	(100, 100)	0.176	0.165	0.161	0.161	0.232	0.232
	(50, 300)	0.111	0.104	0.103	0.103	0.152	0.152
(16.50, 7.00)	(20, 30)	0.155	0.136	0.129	0.131	0.200	0.200
	(30, 30)	0.148	0.143	0.141	0.142	0.192	0.192
	(50, 50)	0.138	0.126	0.124	0.124	0.181	0.181
	(50, 100)	0.121	0.104	0.100	0.101	0.155	0.155
	(100, 100)	0.127	0.121	0.119	0.120	0.169	0.169
	(50, 300)	0.109	0.095	0.094	0.093	0.142	0.142
(16.50, 7.00)	(20, 30)	0.120	0.108	0.106	0.106	0.159	0.159
	(30, 30)	0.108	0.091	0.088	0.089	0.136	0.136
	(50, 50)	0.105	0.100	0.098	0.099	0.136	0.136
	(50, 100)	0.098	0.087	0.086	0.086	0.128	0.128
	(100, 100)	0.086	0.073	0.071	0.071	0.111	0.111
	(50, 300)	0.102	0.082	0.080	0.080	0.121	0.121

**Table 24.** Estimated average lengths of the confidence intervals for  $\text{sens}(c_s)$ , when  $X \sim 0.75\text{Beta}(1, 3) + 0.25\text{Beta}(5, 1.75)$  and  $Y \sim 0.75\text{Beta}(\alpha_{Y_1}, 2) + 0.25\text{Beta}(\alpha_{Y_2}, 4.5)$ .

Scenario ( $\alpha_{Y_1}, \alpha_{Y_2}$ )	Sample sizes ( $n_1, n_2$ )	Methods					
		$\delta$ -BC	BC-AN	BC-PB	BC-bias	EL	ECS
(16.50, 7.00)	(20, 30)	0.221	0.253	0.245	0.246	0.281	0.281
	(30, 30)	0.206	0.243	0.227	0.229	0.257	0.257
	(50, 50)	0.183	0.200	0.176	0.181	0.221	0.221
	(50, 100)	0.193	0.216	0.212	0.212	0.244	0.244
	(100, 100)	0.180	0.206	0.197	0.198	0.226	0.226
	(50, 300)	0.123	0.136	0.134	0.134	0.163	0.163
(16.50, 7.00)	(20, 30)	0.158	0.163	0.151	0.153	0.198	0.198
	(30, 30)	0.150	0.165	0.162	0.162	0.190	0.190
	(50, 50)	0.141	0.158	0.154	0.154	0.176	0.176
	(50, 100)	0.124	0.124	0.119	0.120	0.154	0.154
	(100, 100)	0.136	0.148	0.146	0.146	0.174	0.174
	(50, 300)	0.113	0.136	0.132	0.132	0.145	0.145
(16.50, 7.00)	(20, 30)	0.128	0.146	0.142	0.142	0.158	0.158
	(30, 30)	0.114	0.121	0.115	0.117	0.137	0.137
	(50, 50)	0.107	0.116	0.114	0.114	0.134	0.134
	(50, 100)	0.100	0.111	0.109	0.110	0.124	0.124
	(100, 100)	0.088	0.088	0.086	0.086	0.109	0.109
	(50, 300)	0.098	0.126	0.120	0.120	0.125	0.125

## Scenarios for the Simulation Study of Section 3.2

**Table 25.** Different scenarios considered in the *second* simulation study.

	Distribution of $X$	Distribution of $Y$
(1a)	$X \sim N(\mu_X = 0, \sigma_X^2 = 1)$	$Y \sim N(\mu_Y, \sigma_Y^2 = 0.25)$
(1b)	$X \sim N(\mu_X = 6.5, \sigma_X^2 = 0.09)$	$Y \sim N(\mu_Y, \sigma_Y^2 = 0.25)$
(2)	$\log(X) \sim N(\mu_X = 2.5, \sigma_X^2 = 0.09)$	$\log(Y) \sim N(\mu_Y, \sigma_Y^2 = 0.25)$
(3)	$X \sim \text{Beta}(1, 3)$	$Y \sim \text{Beta}(\gamma, 1/3)$
(4a)	$X \sim \text{Exp}(1)$	$Y \sim \text{Exp}(\gamma)$
(4b)	$X \sim \text{Gamma}(\alpha_X = 2, \beta_X = 2)$	$Y \sim \text{Gamma}(\alpha_Y = 2, \beta_Y)$
(5a)	$X \sim \frac{1}{2}N(\mu_{X_1} = 10, \sigma_{X_1}^2 = 1) + \frac{1}{2}N(\mu_{X_2} = 13, \sigma_{X_2}^2 = 1)$	$Y \sim \frac{1}{2}N(\mu_{Y_1}, \sigma_{Y_1}^2 = 1) + \frac{1}{2}N(\mu_{Y_2}, \sigma_{Y_2}^2 = 5)$
(5b)	$X \sim \frac{3}{4}\text{Beta}(\alpha_{X_1} = 1, \beta_{X_1} = 3) + \frac{1}{4}\text{Beta}(\alpha_{X_2} = 5, \beta_{X_2} = 1.75)$	$Y \sim \frac{3}{4}\text{Beta}(\alpha_{Y_1}, \beta_{Y_1} = 2) + \frac{1}{4}\text{Beta}(\alpha_{Y_2}, \beta_{Y_2} = 4.5)$

## Results from the Simulation Study of Section 3.2

**Table 26.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim N(\mu_X = 0, \sigma_X^2 = 1)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.25)$ .

$c_s$	$\mu_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
					0.90	0.95	0.99	0.90	0.95	0.99
0.524	0.786	0.7	20	20	0.880	0.950	0.990	0.899	0.952	0.990
			50	20	0.887	0.947	0.989	0.904	0.945	0.990
			20	50	0.884	0.942	0.988	0.899	0.941	0.988
			50	50	0.896	0.95	0.989	0.894	0.951	0.988
			100	100	0.900	0.948	0.990	0.890	0.951	0.989
0.842	1.263	0.8	20	20	0.899	0.940	0.973	0.903	0.94	0.971
			50	20	0.898	0.946	0.982	0.900	0.949	0.980
			20	50	0.896	0.945	0.980	0.898	0.946	0.980
			30	30	0.892	0.943	0.990	0.901	0.949	0.988
			50	50	0.896	0.945	0.989	0.892	0.943	0.988
1.282	1.923	0.9	100	100	0.894	0.949	0.990	0.900	0.947	0.990
			50	50	0.877	0.945	0.983	0.918	0.953	0.981
			50	75	0.887	0.941	0.987	0.905	0.949	0.984
			75	50	0.883	0.938	0.985	0.902	0.948	0.983
			100	100	0.894	0.944	0.989	0.911	0.956	0.990

**Table 27.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim N(\mu_X = 6.5, \sigma_X^2 = 0.09)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.25)$ .

$c_s$	$\mu_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
					0.90	0.95	0.99	0.90	0.95	0.99
6.657	6.919	0.7	20	20	0.884	0.952	0.991	0.901	0.954	0.990
			50	20	0.889	0.946	0.990	0.903	0.944	0.990
			20	50	0.891	0.948	0.992	0.903	0.947	0.990
			50	50	0.896	0.948	0.990	0.892	0.947	0.988
			100	100	0.905	0.951	0.991	0.896	0.953	0.991
6.752	7.172	0.8	20	20	0.901	0.941	0.972	0.901	0.942	0.970
			50	20	0.894	0.946	0.982	0.892	0.949	0.979
			20	50	0.893	0.943	0.981	0.895	0.946	0.980
			30	30	0.895	0.944	0.988	0.905	0.949	0.986
			50	50	0.900	0.951	0.99	0.899	0.945	0.989
6.884	7.524	0.9	100	100	0.895	0.945	0.988	0.898	0.943	0.989
			50	50	0.872	0.944	0.984	0.917	0.950	0.982
			50	75	0.891	0.943	0.988	0.903	0.951	0.985
			75	50	0.892	0.944	0.987	0.907	0.949	0.984
			100	100	0.901	0.954	0.989	0.919	0.961	0.990

**Table 28.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $\log(X) \sim N(\mu_X = 2.5, \sigma_X^2 = 0.09)$  and  $\log(Y) \sim N(\mu_Y, \sigma_Y^2 = 0.25)$ .

$c_s$	$\mu_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
					0.90	0.95	0.99	0.90	0.95	0.99
14.258	2.920	0.7	20	20	0.883	0.956	0.990	0.903	0.956	0.990
			50	20	0.886	0.945	0.987	0.901	0.946	0.988
			20	50	0.885	0.945	0.992	0.898	0.943	0.990
			50	50	0.891	0.947	0.990	0.888	0.947	0.989
			100	100	0.900	0.950	0.988	0.891	0.952	0.989
15.682	3.173	0.8	20	20	0.900	0.942	0.971	0.899	0.939	0.968
			50	20	0.894	0.943	0.980	0.896	0.946	0.977
			20	50	0.894	0.946	0.981	0.897	0.949	0.978
			30	30	0.893	0.945	0.989	0.903	0.949	0.988
			50	50	0.895	0.947	0.991	0.892	0.944	0.990
17.894	3.525	0.9	100	100	0.901	0.954	0.990	0.905	0.952	0.991
			50	50	0.876	0.947	0.984	0.920	0.952	0.979
			50	75	0.885	0.942	0.987	0.903	0.947	0.983
			75	50	0.889	0.944	0.988	0.907	0.952	0.984
			100	100	0.899	0.949	0.989	0.915	0.957	0.988

**Table 29.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \text{Beta}(1, 3)$  and  $Y \sim \text{Beta}(\gamma, 1/3)$ .

$c_s$	$\mu_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
					0.90	0.95	0.99	0.90	0.95	0.99
0.331	0.496	0.7	20	20	0.875	0.948	0.990	0.897	0.952	0.990
			50	20	0.886	0.945	0.989	0.903	0.946	0.989
			20	50	0.887	0.948	0.990	0.902	0.944	0.990
			50	50	0.890	0.946	0.989	0.888	0.945	0.988
			100	100	0.909	0.953	0.990	0.899	0.956	0.991
0.416	0.857	0.8	20	20	0.901	0.942	0.970	0.899	0.94	0.969
			50	20	0.892	0.943	0.979	0.891	0.947	0.978
			20	50	0.895	0.945	0.982	0.896	0.946	0.980
			30	30	0.893	0.945	0.991	0.906	0.951	0.989
			50	50	0.897	0.948	0.991	0.895	0.946	0.991
			100	100	0.900	0.951	0.988	0.903	0.948	0.989
0.536	1.848	0.9	50	50	0.879	0.948	0.983	0.922	0.955	0.981
			50	75	0.882	0.937	0.985	0.898	0.945	0.982
			75	50	0.888	0.941	0.984	0.906	0.947	0.982
			100	100	0.893	0.946	0.988	0.908	0.957	0.990

**Table 30.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \text{Exp}(1)$  and  $Y \sim \text{Exp}(\gamma)$ .

$c_s$	$\gamma$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
					0.90	0.95	0.99	0.90	0.95	0.99
1.204	0.296	0.7	20	20	0.878	0.950	0.991	0.901	0.952	0.989
			50	20	0.894	0.951	0.989	0.906	0.950	0.991
			20	50	0.886	0.950	0.989	0.902	0.947	0.991
			50	50	0.891	0.950	0.992	0.887	0.948	0.990
			100	100	0.902	0.950	0.989	0.893	0.953	0.989
1.609	0.138	0.8	20	20	0.904	0.943	0.973	0.903	0.941	0.971
			50	20	0.901	0.948	0.981	0.901	0.951	0.980
			20	50	0.894	0.944	0.979	0.895	0.948	0.978
			30	30	0.892	0.943	0.991	0.902	0.950	0.988
			50	50	0.895	0.947	0.991	0.894	0.945	0.991
			100	100	0.903	0.950	0.989	0.906	0.951	0.989
2.303	0.046	0.9	50	50	0.872	0.943	0.984	0.916	0.951	0.980
			50	75	0.888	0.942	0.986	0.908	0.950	0.982
			75	50	0.883	0.939	0.986	0.900	0.946	0.982
			100	100	0.900	0.953	0.988	0.917	0.960	0.990



**Table 31.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \text{Gamma}(\alpha_X = 2, \beta_X = 2)$  and  $Y \sim \text{Gamma}(\alpha_Y = 2, \beta_Y)$ .

$c_s$	$\beta_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
					0.90	0.95	0.99	0.90	0.95	0.99
4.878	4.446	0.7	20	20	0.884	0.954	0.990	0.904	0.957	0.990
				50	0.889	0.952	0.990	0.906	0.948	0.990
			50	20	0.892	0.948	0.989	0.906	0.948	0.990
				50	0.893	0.950	0.990	0.888	0.949	0.990
			100	100	0.905	0.953	0.991	0.895	0.955	0.991
5.988	7.264	0.8	20	20	0.900	0.942	0.971	0.903	0.942	0.969
				50	0.896	0.942	0.977	0.896	0.947	0.976
			50	20	0.893	0.943	0.981	0.894	0.946	0.979
				30	0.894	0.947	0.991	0.907	0.954	0.989
			100	100	0.901	0.947	0.992	0.898	0.942	0.991
7.779	14.628	0.9	50	50	0.879	0.949	0.986	0.922	0.956	0.980
				75	0.891	0.943	0.988	0.905	0.951	0.985
			75	50	0.890	0.942	0.986	0.908	0.952	0.984
			100	100	0.891	0.948	0.990	0.910	0.956	0.990

**Table 32.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \frac{1}{2}N(\mu_{X_1} = 10, \sigma_{X_1}^2 = 1) + \frac{1}{2}N(\mu_{X_2} = 13, \sigma_{X_2}^2 = 1)$  and  $Y \sim \frac{1}{2}N(\mu_{Y_1}, \sigma_{Y_1}^2 = 1) + \frac{1}{2}N(\mu_{Y_2}, \sigma_{Y_2}^2 = 5)$ .

$c_s$	$\mu_{Y_1}$	$\mu_{Y_2}$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
						0.90	0.95	0.99	0.90	0.95	0.99
12.73	12.580	16.580	0.7	20	20	0.888	0.955	0.990	0.906	0.958	0.991
					50	0.886	0.946	0.988	0.900	0.942	0.988
				50	20	0.884	0.943	0.986	0.894	0.94	0.987
					50	0.888	0.946	0.987	0.883	0.944	0.986
				100	100	0.900	0.947	0.990	0.888	0.948	0.988
13.27	13.607	17.606	0.8	20	20	0.900	0.942	0.970	0.903	0.941	0.968
					50	0.897	0.946	0.981	0.903	0.950	0.980
				50	20	0.899	0.947	0.980	0.904	0.949	0.979
					30	0.894	0.942	0.988	0.904	0.950	0.987
				100	100	0.895	0.946	0.991	0.896	0.949	0.992
13.830	14.731	18.732	0.9	50	50	0.876	0.948	0.986	0.920	0.956	0.981
					75	0.892	0.940	0.987	0.906	0.951	0.983
				75	50	0.887	0.939	0.986	0.903	0.948	0.981
				100	100	0.889	0.947	0.988	0.908	0.955	0.987

**Table 33.** Estimated coverage probabilities of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \frac{3}{4}\text{Beta}(\alpha_{X_1} = 1, \beta_{X_1} = 3) + \frac{1}{4}\text{Beta}(\alpha_{X_2} = 5, \beta_{X_2} = 1.75)$  and  $Y \sim \frac{3}{4}\text{Beta}(\alpha_{Y_1}, \beta_{Y_1} = 2) + \frac{1}{4}\text{Beta}(\alpha_{Y_2}, \sigma_{Y_2} = 4.5)$ .

$c_s$	$\mu_{Y_1}$	$\mu_{Y_2}$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ coverages			$R_\alpha^*$ coverages		
						0.90	0.95	0.99	0.90	0.95	0.99
0.532	5.21	2.71	0.7	20	20	0.882	0.953	0.990	0.903	0.956	0.991
				50	20	0.888	0.948	0.989	0.904	0.947	0.990
				20	50	0.887	0.949	0.989	0.904	0.947	0.990
				50	50	0.893	0.946	0.988	0.890	0.947	0.988
				100	100	0.896	0.947	0.990	0.886	0.950	0.990
0.68	15.00	6.40	0.8	20	20	0.897	0.943	0.968	0.907	0.942	0.966
				50	20	0.892	0.943	0.978	0.900	0.948	0.978
				20	50	0.893	0.944	0.980	0.902	0.951	0.980
				30	30	0.898	0.948	0.991	0.913	0.956	0.990
				50	50	0.893	0.949	0.991	0.899	0.950	0.993
0.815	33.52	22.00	0.9	100	100	0.897	0.948	0.987	0.905	0.948	0.990
				50	50	0.876	0.944	0.984	0.918	0.954	0.980
				50	75	0.890	0.944	0.987	0.911	0.954	0.985
				75	50	0.882	0.934	0.984	0.903	0.949	0.981
				100	100	0.901	0.951	0.991	0.920	0.961	0.991

**Table 34.** Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim N(\mu_X = 0, \sigma_X^2 = 1)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.25)$ .

$c_s$	$\mu_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
					0.90	0.95	0.99	0.90	0.95	0.99
0.524	0.786	0.7	20	20	0.141	0.183	0.283	0.141	0.184	0.285
			50	20	0.091	0.118	0.183	0.091	0.119	0.186
			20	50	0.089	0.116	0.178	0.0888	0.115	0.177
			50	50	0.057	0.074	0.115	0.057	0.074	0.115
			100	100	0.029	0.037	0.058	0.029	0.037	0.058
0.842	1.263	0.8	20	20	0.131	0.171	0.257	0.141	0.184	0.275
			50	20	0.086	0.112	0.174	0.090	0.118	0.183
			20	50	0.084	0.110	0.170	0.086	0.113	0.177
			30	30	0.090	0.118	0.183	0.093	0.123	0.193
			50	50	0.054	0.071	0.110	0.055	0.073	0.114
1.282	1.923	0.9	100	100	0.027	0.035	0.055	0.027	0.036	0.056
			50	50	0.049	0.064	0.098	0.053	0.070	0.112
			50	75	0.040	0.052	0.0810	0.043	0.057	0.091
			75	50	0.040	0.0524	0.082	0.043	0.058	0.093
			100	100	0.024	0.032	0.050	0.025	0.034	0.054

**Table 35.** Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim N(\mu_X = 6.5, \sigma_X^2 = 0.09)$  and  $Y \sim N(\mu_Y, \sigma_Y^2 = 0.25)$ .

$c_s$	$\mu_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
					0.90	0.95	0.99	0.90	0.95	0.99
6.657	6.919	0.7	20	20	0.075	0.097	0.147	0.075	0.098	0.148
			50	20	0.048	0.062	0.094	0.048	0.062	0.093
			20	50	0.049	0.064	0.098	0.050	0.065	0.101
			50	50	0.031	0.041	0.062	0.031	0.041	0.062
			100	100	0.016	0.021	0.032	0.016	0.021	0.032
6.752	7.172	0.8	20	20	0.069	0.090	0.133	0.071	0.092	0.135
			50	20	0.044	0.058	0.088	0.046	0.060	0.092
			20	50	0.046	0.060	0.091	0.048	0.063	0.095
			30	30	0.048	0.062	0.0956	0.049	0.0650	0.101
			50	50	0.029	0.038	0.059	0.030	0.039	0.061
6.884	7.524	0.9	100	100	0.015	0.019	0.030	0.015	0.020	0.030
			50	50	0.049	0.064	0.098	0.025	0.034	0.053
			50	75	0.019	0.025	0.039	0.021	0.027	0.044
			75	50	0.019	0.025	0.039	0.020	0.027	0.044
			100	100	0.012	0.016	0.024	0.012	0.016	0.026

**Table 36.** Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $\log(X) \sim N(\mu_X = 2.5, \sigma_X^2 = 0.09)$  and  $\log(Y) \sim N(\mu_Y, \sigma_Y^2 = 0.25)$ .

$c_s$	$\mu_Y$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
					0.90	0.95	0.99	0.90	0.95	0.99
14.258	2.920	0.7	20	20	0.295	0.386	0.598	0.296	0.389	0.606
			50	20	0.184	0.239	0.367	0.182	0.237	0.363
			20	50	0.190	0.249	0.388	0.192	0.253	0.398
			50	50	0.119	0.155	0.239	0.119	0.155	0.239
			100	100	0.059	0.077	0.119	0.059	0.077	0.119
15.682	3.173	0.8	20	20	0.367	0.480	0.735	0.386	0.509	0.770
			50	20	0.228	0.297	0.460	0.233	0.305	0.479
			20	50	0.239	0.315	0.495	0.253	0.338	0.533
			30	30	0.246	0.324	0.508	0.256	0.341	0.544
			50	50	0.149	0.194	0.302	0.152	0.200	0.316
17.894	3.525	0.9	100	100	0.074	0.097	0.150	0.075	0.098	0.153
			50	50	0.187	0.246	0.382	0.338	0.424	0.581
			50	75	0.153	0.203	0.322	0.168	0.227	0.370
			75	50	0.150	0.197	0.308	0.160	0.214	0.349
			100	100	0.092	0.121	0.191	0.097	0.129	0.210

**Table 37.** Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \text{Beta}(1, 3)$  and  $Y \sim \text{Beta}(\gamma, 1/3)$ .

$c_s$	$\gamma$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
					0.90	0.95	0.99	0.90	0.95	0.99
0.331	0.496	0.7	20	20	0.079	0.103	0.159	0.079	0.104	0.160
			50	20	0.050	0.065	0.100	0.050	0.065	0.100
			20	50	0.051	0.066	0.102	0.051	0.067	0.103
			50	50	0.032	0.042	0.065	0.032	0.042	0.065
			100	100	0.016	0.021	0.032	0.016	0.021	0.032
0.416	0.857	0.8	20	20	0.074	0.097	0.145	0.078	0.102	0.155
			50	20	0.047	0.062	0.095	0.048	0.064	0.099
			20	50	0.048	0.063	0.097	0.050	0.066	0.102
			30	30	0.050	0.066	0.102	0.052	0.069	0.108
			50	50	0.030	0.040	0.062	0.031	0.041	0.064
0.536	1.848	0.9	100	100	0.015	0.020	0.031	0.015	0.020	0.031
			50	50	0.027	0.036	0.056	0.028	0.036	0.056
			50	75	0.022	0.029	0.046	0.024	0.032	0.052
			75	50	0.022	0.029	0.045	0.024	0.032	0.051
			100	100	0.014	0.018	0.028	0.014	0.019	0.030

**Table 38.** Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \text{Exp}(1)$  and  $Y \sim \text{Exp}(\gamma)$ .

$c_s$	$\gamma$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
					0.90	0.95	0.99	0.90	0.95	0.99
1.204	0.296	0.7	20	20	1.146	1.494	2.311	1.146	1.503	2.334
			50	20	0.716	0.931	1.434	0.712	0.927	1.421
			20	50	0.730	0.951	1.483	0.739	0.971	1.519
			50	50	0.463	0.603	0.931	0.463	0.603	0.930
			100	100	0.232	0.302	0.466	0.232	0.302	0.465
1.609	0.138	0.8	20	20	1.144	1.493	2.307	1.214	1.584	2.396
			50	20	0.746	0.971	1.501	0.762	1.000	1.563
			20	50	0.768	1.009	1.574	0.804	1.067	1.675
			30	30	0.7618	0.994	1.539	0.825	1.095	1.735
			50	50	0.481	0.627	0.975	0.491	0.645	1.015
2.303	0.046	0.9	100	100	0.242	0.315	0.487	0.243	0.318	0.495
			50	50	0.495	0.649	1.009	0.537	0.721	1.161
			50	75	0.406	0.534	0.841	0.439	0.589	0.960
			75	50	0.403	0.528	0.824	0.427	0.568	0.920
			100	100	0.248	0.325	0.506	0.257	0.341	0.549

**Table 39.** Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \text{Gamma}(\alpha_X = 2, \beta_X = 2)$  and  $Y \sim \text{Gamma}(\alpha_Y = 2, \beta_Y)$ .

$c_s$	$\gamma$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
					0.90	0.95	0.99	0.90	0.95	0.99
4.878	4.446	0.7	20	20	0.780	1.017	1.571	0.783	1.026	1.589
				50	0.493	0.640	0.985	0.492	0.639	0.980
			50	20	0.504	0.656	1.020	0.502	0.659	1.028
				50	0.318	0.414	0.638	0.316	0.412	0.636
			100	100	0.160	0.208	0.321	0.160	0.208	0.320
5.988	7.264	0.8	20	20	0.907	1.186	1.827	0.902	1.179	1.802
				50	0.569	0.742	1.145	0.579	0.759	1.185
			50	20	0.584	0.768	1.201	0.613	0.813	1.280
				30	0.606	0.794	1.240	0.628	0.833	1.319
			100	100	0.368	0.480	0.746	0.373	0.490	0.771
7.779	14.628	0.9	50	50	0.184	0.240	0.371	0.186	0.243	0.378
				75	0.426	0.559	0.868	0.426	0.559	0.868
			75	50	0.348	0.459	0.724	0.379	0.511	0.833
				100	0.344	0.451	0.704	0.365	0.487	0.790
			100	100	0.211	0.277	0.433	0.221	0.294	0.474

**Table 40.** Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \frac{1}{2}N(\mu_{X_1} = 10, \sigma_{X_1}^2 = 1) + \frac{1}{2}N(\mu_{X_2} = 13, \sigma_{X_2}^2 = 1)$  and  $Y \sim \frac{1}{2}N(\mu_{Y_1}, \sigma_{Y_1}^2 = 1) + \frac{1}{2}N(\mu_{Y_2}, \sigma_{Y_2}^2 = 5)$ .

$c_s$	$\mu_{Y_1}$	$\mu_{Y_2}$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
						0.90	0.95	0.99	0.90	0.95	0.99
12.73	12.580	16.580	0.7	20	20	0.364	0.476	0.740	0.367	0.483	0.756
					50	0.231	0.301	0.468	0.231	0.304	0.475
				50	20	0.231	0.302	0.468	0.231	0.303	0.473
					50	0.146	0.191	0.295	0.146	0.191	0.296
				100	100	0.073	0.095	0.147	0.073	0.095	0.147
13.27	13.607	17.606	0.8	20	20	0.284	0.373	0.573	0.303	0.401	0.616
					50	0.190	0.252	0.401	0.189	0.251	0.399
				50	20	0.181	0.238	0.373	0.189	0.250	0.397
					30	0.200	0.266	0.427	0.199	0.265	0.425
				100	100	0.115	0.151	0.235	0.118	0.155	0.246
13.830	14.731	18.732	0.9	50	50	0.058	0.075	0.116	0.058	0.076	0.119
					75	0.090	0.119	0.185	0.099	0.133	0.216
				75	50	0.075	0.099	0.155	0.080	0.107	0.175
					100	0.075	0.099	0.155	0.080	0.108	0.176
				100	100	0.046	0.060	0.094	0.048	0.063	0.102

**Table 41.** *Estimated areas of the confidence regions for  $(c_s, \text{sens}(c_s))$ , when  $X \sim \frac{3}{4}\text{Beta}(\alpha_{X_1} = 1, \beta_{X_1} = 3) + \frac{1}{4}\text{Beta}(\alpha_{X_2} = 5, \beta_{X_2} = 1.75)$  and  $Y \sim \frac{3}{4}\text{Beta}(\alpha_{Y_1}, \beta_{Y_1} = 2) + \frac{1}{4}\text{Beta}(\alpha_{Y_2}, \sigma_{Y_2} = 4.5)$ .*

$c_s$	$\mu_{Y_1}$	$\mu_{Y_2}$	$\text{sens}(c_s)$	$m$	$n$	$R_\alpha$ areas			$R_\alpha^*$ areas		
						0.90	0.95	0.99	0.90	0.95	0.99
0.532	5.21	2.71	0.7	20	20	0.073	0.094	0.142	0.073	0.094	0.141
				50	20	0.048	0.062	0.094	0.048	0.062	0.094
				20	50	0.047	0.061	0.092	0.047	0.060	0.091
				50	50	0.031	0.040	0.061	0.031	0.040	0.061
				100	100	0.016	0.020	0.031	0.016	0.020	0.031
0.68	15.00	6.40	0.8	20	20	0.061	0.079	0.115	0.064	0.083	0.120
				50	20	0.041	0.053	0.081	0.043	0.056	0.085
				20	50	0.040	0.052	0.078	0.041	0.053	0.081
				30	30	0.043	0.056	0.085	0.045	0.058	0.089
				50	50	0.027	0.035	0.053	0.027	0.036	0.055
0.815	33.52	22.00	0.9	100	100	0.014	0.018	0.028	0.014	0.018	0.028
				50	50	0.012	0.015	0.023	0.013	0.017	0.027
				50	75	0.010	0.012	0.019	0.010	0.014	0.022
				75	50	0.010	0.013	0.020	0.011	0.014	0.023
				100	100	0.006	0.008	0.012	0.006	0.008	0.013